Functional Skills Maths

Activity Bank

OCR
RECOGNISING ACHIEVEMENT

Contents

Functional Skills Maths

Functional skills qualification in Maths at Level 1 and Level 2

Geometry

Which way to roll? (Entry 2 – Level 1)

Introduction

This activity introduces area and volume with a problem to solve and justify. The answer can be found using a practical method; the functionality comes in when you ask how and why.
It is in no fixed context or vocational area so should be applicable to all learners and present an everyday problem that could be related to many scenarios.

It incorporates assumption and planning but also gives an opportunity to use measurement, estimation and formula and using checking by practical application.

Once completed the skills and processes practiced can be identified and discussed to see where they might be useful in contexts related to the learners.

I have used this task as an activity after I have taught measurement using metric units of length, but often before I have introduced the concept of area and volume.

The numeracy skills within this task include estimation, measuring with cm, mm, mm^2 and cm^2 and possibly litres. Volume measure could include mm^3, cm^3, litres, ml and cl.
Graphs and charts could also be introduced to make predictions.

If the task was kept to a practical investigation and then results put straight into a chart, the task would be suitable for entry level 2 or 3. If it were simply the investigation on a practical level, it would be suitable for entry level 1.

Moving it up to mathematically calculating the volume and investigating the relationship between volume and surface area would increase the task to level 1 and possibly level 2 if you looked at percentage increase.

Some of the useful functional skills that this can be used to highlight and practice are:

- Making a plan of a task – deciding how to check the decision, what maths to do, how to present the results etc - Representing
- Deciding on levels of accuracy required and affect of rounding on the outcome – Analysis of the data
- Assumptions – eg would it be practical to have a cylinder 1 metre high but only 2cm radius? – Interpretation of the answer
- Is there a point at which you could not perform the practical task, say because the contents were individually too large to fit in the cylinder?

To make sure you are practising the functionality of the learners' maths skills, the numeracy skills required should be checked before hand to make sure a weakness here is not a barrier to tackling the problem.

Key stage 4 references

FA1 General problem solving skills	
1.1 – Solve problems using mathematical skills	Candidates should be able to: a) select and use suitable problem solving strategies and efficient techniques to solve numerical problems; b) identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches; c) break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods; d) use notation and symbols correctly and consistently within a problem; e) use a range of strategies to create numerical representations of a problem and its solution; move from one form of representation to another in order to get different perspectives on the problem; f) interpret and discuss numerical information presented in a variety of forms; g) present and interpret solutions in the context of the original problem; h) review and justify their choice of mathematical presentation; i) understand the importance of counter-example and identify exceptional cases when solving problems; j) show step-by-step deduction in solving a problem; k) recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying those assumptions may have on the solution to a problem.

FA9 General measures	
9.1 – Interpret scales and use measurements	Candidates should be able to: a) interpret scales on a range of measuring instruments, including those for time and mass; b) know that measurements using real numbers depend on the choice of unit; c) understand angle measure using the associated language; d) make sensible estimates of a range of measures in everyday settings; e) convert measurements from one unit to another; f) know rough metric equivalents of pounds, feet, miles, pints and gallons.

Resources needed

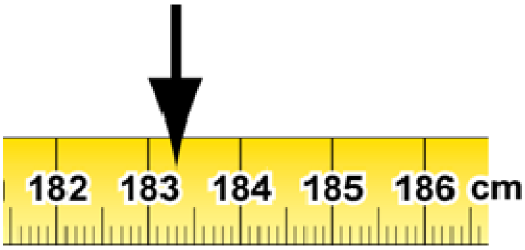
Calculators, rulers, pencils, A4 paper, scissors, cereals or rice, sticky tape, graph paper.

Misconceptions and common errors

Weaker learners may not see the process of planning the task as important.
Visualisation of amounts – this will become clear when estimating amounts.
Confusion switching between units; mixing units in calculations.
Reading scales accurately / correctly.
Using a given formula; substituting and manipulation.

Task 3 – Body Mass Index

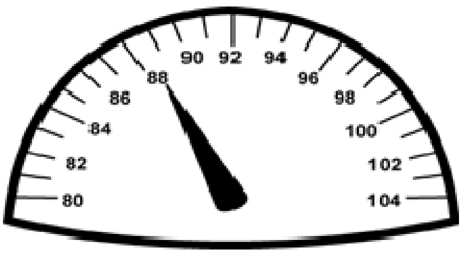
Brian wants to know his Body Mass Index (BMI).
He needs his height and weight.
He uses a tape measure and weighing machine.
He marks the tape to show his height.



(a) What is Brian's height to the nearest centimetre?

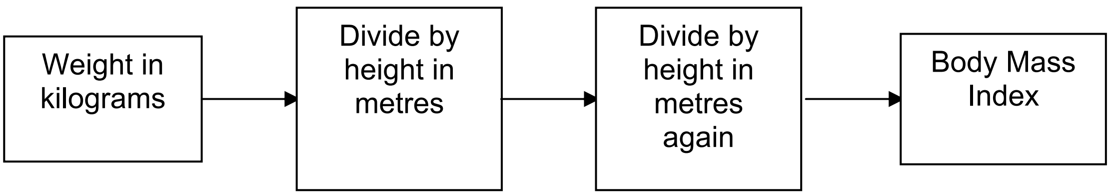
Now Brian weighs himself.

(b) What is Brian's weight in kilograms?



............kg

(c) Brian finds this method of working out his BMI.



Work out Brian's BMI.

The Principal Examiner commented:

Task 3:

Part (a) Usually correct, though 183.3 was common and 186 a strange outsider.

Part (b) Usually correct.

Part (c) This was often well answered, although many wrote an answer only and lost "Representing" marks. Frequent errors were, not to convert the measurement in centimetres into metres or to only complete one of the divisions.

Part (d) The general level of responses here were disappointing because a common response was to write 24.9 alone, or 84.5 alone, with no further comment. This was disappointing because the candidates had clearly adopted some sort of method but had not shown it. Many candidates lost "Representing", "Analysing" and "Interpreting" marks in this way.
Some candidates reversed the flow chart and scored full marks easily.
Some candidates realised that they could use trial and improvement and began working towards a BMI of 25. They also scored full marks with a little more work.
A few candidates wrote, "He should aim for 24" or gave a lengthy discourse on healthy eating!

Main Activity: Which way to roll?

So which tube holds the most?

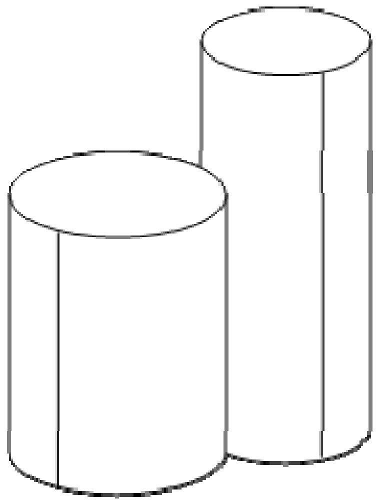
You have an A4 piece of paper.

You could make a cylinder by rolling it in two different ways.

Which would hold more?

WHY??

- Before you start have a guess and record everyone's decision
- Next you need to plan how you will go about solving the problem.
- Make sure your plan shows the correct order of operation, includes any assumptions you have to make and don't forget to write any hints to yourself so you don't forget things as you work through the task.
- When you are happy that your plan is complete, check it through with your tutor / teacher and then you can start the task.
- As you work through, make sure you write the measurements and units carefully and explain any calculations you do so you can describe your methods to someone afterwards.



When you have finished…

- Were you right?
- How could you prove it?
- Is there a different way of solving the problem?

Lesson plan: Which way to roll?

Time	Content
5 - 10 minutes	Review understanding of measurement – confirm learners are able to measure in millimetres, centimetres and metres and convert between the three units. Review understanding of area and volume – including the importance of using like units.
5 - 10 minutes	Hand out a clean piece of A4 paper and explain that it can be rolled into a cylinder two ways. One taller that the other. Ask the question: Which will hold more? Give a few minutes for discussion between learners and record choices. (Many learners will say it's the same as the area of the paper is the same) Discuss what information might be needed to solve the problem and how this could be collected. (Could be a chance here to ask how to record choices; tally, table, is there an even split?) Decide on what equipment will be needed to obtain the measurements. Discuss and agree a plan of how to complete task. This stage may be done as a group discussion or could be done in small groups or pairs with the groups feeding back. This then elicits comparisons, promotes constructive criticism and may highlight a variation in methods and accuracy. Ask how they could check to see if their decision is correct? Suggest the practical route.
10 – 15 minutes	Hand out tape and ask learners in small groups or pairs to make their cylinders. If you have enough cereal or rice, they could attempt to try comparing the two – Warning – this can get messy! If not, demonstrate. HINT – place the taller cylinder inside the shorter one. Fill the taller one then remove it from the shorter one. Compare the result with those collected. This is a good point to ask why and see if anyone can come up with a mathematical justification as to why the shorter cylinder holds more.
20 - 25 minutes	Ask learners to measure the height of the cereal or contents in the short tube and be prepared to record the data. Now ask what would happen if they took another piece of A4 paper, cut it in half long ways, taped up the short edges and made this into a cylinder. Try the same thing by cutting another A4 sheet in half the short way, tape up the shorter edges and make this into a cylinder. In both cases measure the height of the columns of cereals. This can be done again making one twice as tall, another half the height. Ask the questions, what is happening and how could you explain it mathematically?
10 minutes	Draw the class together and compare results and methods. Explain the mathematical properties of a sphere and that although the paper has the same surface area, the circles at the top and bottom are getting progressively larger as the tube gets shorter, hence an increase in volume.

This extension is optional.
It could well be used as another session to follow on

Time	Content
10 - 15 minutes	If this is a level 1 or 2 group of learners you can now go on to look at calculating the volume by measuring the circumference, substituting it into the area formula and then calculating the volume.
25–30 minutes	Depending on the level of the group you could also now look at plotting the results on a line graph and seeing if there is a relationship between height, radius, circumference and volume. With this data, predictions can be made and results interpreted.

Column of pounds (Entry 3 – Level 1)

Introduction

The aim of this activity is to give learners practice in problem solving using a range of measuring skills including weight and length in a very practical way.

It will also be an opportunity to incorporate assumptions and planning skills.

Being in no particular context it should be applicable to all learners but could be contextualised as a further activity. This also means that once completed the skills and processes practiced within the task can be identified and discussed to see where they might be used in other, more practical situations.

I have used this task as an activity during the teaching of measurement to reinforce the conversion between units of measure and to bring in a practical context. I often have the learners decide on something other than coins to compare like £5 notes or stamps.

The numeracy skills in this task are level 1 but even entry 3 learners should, with a little support, be able to complete this task. I have used it in a mixed group with the stronger learners paired up and given further challenges on top such as what height / weight would someone need to be to make it equal?

Arranging the learners into pairs or small groups and asking them to write their annotated calculations and results onto flip chart paper and explaining this to the group, will help them to get into good habits when answering assessment papers.

Some of the useful functional skills that this can be used to highlight and practice are:

- making a plan of a task – Representing
- deciding on levels of accuracy required and affect of rounding on the outcome – Analysis of the data
- assumptions – eg could you pile coins that height without them falling over
- annotating the calculations to show how the maths solves the problem
- communicating the process to others.

To make sure you are practising the functionality of the learners' maths skills, the numeracy skills required should be checked before hand to make sure a weakness here is not a barrier to tackling the problem, or be prepared to support the maths so the problem solving is the challenge.

Key stage 4 references

FA1 General problem solving skills	
1.1 – Solve problems using mathematical skills	Candidates should be able to: a) select and use suitable problem solving strategies and efficient techniques to solve numerical problems; b) identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches; c) break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods; d) use notation and symbols correctly and consistently within a problem; e) use a range of strategies to create numerical representations of a problem and its solution; move from one form of representation to another in order to get different perspectives on the problem; f) interpret and discuss numerical information presented in a variety of forms; g) present and interpret solutions in the context of the original problem; h) review and justify their choice of mathematical presentation; i) understand the importance of counter-example and identify exceptional cases when solving problems; j) show step-by-step deduction in solving a problem; k) recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying those assumptions may have on the solution to a problem.

FA9 General measures	
9.1 – Interpret scales and use measurements	Candidates should be able to: a) interpret scales on a range of measuring instruments, including those for time and mass; b) know that measurements using real numbers depend on the choice of unit; c) understand angle measure using the associated language; d) make sensible estimates of a range of measures in everyday settings; e) convert measurements from one unit to another; f) know rough metric equivalents of pounds, feet, miles, pints and gallons.

Resources needed

Calculators, rulers, tape measures, accurate scales, flipchart paper.

Misconceptions and common errors

Weaker learners may not see the process of planning the task as necessary or useful.
Visualisation of very large amounts – this will become clear when choosing the weight or column.
Confusion between switching from units.
Reading various measuring tools and scales accurately/correctly.

Previously set Functional Maths Question (extract)
Level 1 May 2009

Task 3 – Charity Money

Jack and Mary are counting money that has been raised for a local charity.

They have sorted 1p, 2p, 5p, 10p, 20p and 50p coins into piles of bronze and silver coins.

(a) Which coins are bronze and which coins are silver?

(2 marks)

Instead of counting the number of coins in each pile, Mary says that they can weigh each pile of coins. They can then find the number of coins in each pile by using this table that she found on the Internet:

Coin	Weight in grams
£1	9.50
50p	8.00
20p	5.00
10p	6.50
5p	3.25
2p	7.13
1p	3.56

Mary and Jack weigh the coins they have sorted.

They use kitchen scales that can only weigh to the nearest gram.

(b) Explain why this may cause a problem.

(2 marks)

Mary and Jack have 890 g of 1p coins, 827 g of 2p coins, 114 g of 5p coins and 455 g of 10p coins. They also have 84 fifty pence coins and 53 one pound coins.

(c) Work out how much money has been raised and complete the paying in slip needed by the bank. Remember to show all of your working.

BANK PAYING IN SLIP

Coins	Amount	
	Pounds	*Pence*
£2		
£1	53	00
50p & 20p		
10p & 5p		
2p & 1p		
TOTAL CASH		
TOTAL CHEQUES		
£		

(12 marks)

The Principal Examiner commented:

Task 3: Charity Money

3a Most, though not all, scored 2 marks.

3b Most, though not all, scored 1 mark either for mentioning “lack of accuracy in weighing” or “incorrect amount of money”. Few described how the former could result in the latter.

Some mentioned that scales were for weighing, for example, flour not money or that people might read the scales incorrectly (human error) and failed to score.

3c Candidates who “got into” this question often scored a minimum of half marks. Many showed (or, at least performed) the division of mass of coins by mass of a single coin with the intention of finding the number of each denomination of coin. A considerable number then made the error of writing pence after the result and considered they had found the *value* of the coins. Some recovered this by working out the correct values.

Most candidates were able to transfer some reasonable values into the paying-in slip.

Candidates for whom this level was inappropriate either made no attempt to answer the question or thought that the mass of the coins was equivalent to the value of the coins and transferred these numbers into the table.

2 Guidance for Centres

Candidates should:

a) Consider ways of extracting the key data from problems (eg, tabulation, highlighting) so that the route through a problem becomes clearer
b) Practise solving everyday problems and discuss their methods and solutions.
c) Use a calculator
d) Be taught ways to plan a solution (using the planning sheet)
e) Be taught to annotate solutions to problems
f) Write complete, but short, answers that refer to the values drawn or calculated
g) Check that answers are reasonable.

Main Activity: Column of pounds

How much is someone worth?

Would you rather have a column of £1 coins as tall as the headmaster, or a collection of 5p pieces as heavy as him?

<u>Plan</u>

What is the problem I am trying to solve?

What information do I need to find?

What is the maths I need to do?

What form will my answer be?

What assumptions have I made?

Could I have solved the problem in another way?

Where would I use the same sort of skills?

Lesson plan: Column of pounds

Time	Content
5 - 10 minutes	Review understanding of measurement – confirm learners are comfortable using a range of measuring tools such as rulers, tapes and tape measures and scales. Also that they are able to convert between units of length and weight – possibly imperial and metric. Review understanding of area and measurement – including the importance of using the correct units and level of accuracy required for the task.
15 minutes	Introduce the task – Would you rather have a column of £ coins as tall as 'whoever' or the weight of them in 5p pieces? *You could use yourself, another tutor, someone in the class, but obviously be aware of sensitivity – I have done this with learners in pairs using each other but it needs to be carefully thought out.* Collect some guesses and record them – *try to ask the questions, why? how? To elicit some justification behind the suggestions*. Ask: how could we find out which is the greater in value and what information do we need? – *I usually get the learners into pairs or small groups here to work independently with support and encouragement as they develop their ideas and record them as part of their plan*. Ask: How will we get this information? What equipment do we need? Ask the pairs/groups to write up a plan with the calculations they need to do to reach a conclusion.
20 minutes	In their groups, the learners can now start to follow their plan, weighing the person and the 5p and measuring a £ coin and the height of the person. ***It may be helpful to know that the thickness of a £1 coin is 3.15mm, and a 5p coin weighs 3.25g*** Learners should follow their plan, adapting it if necessary, and do the required mathematics to find two amounts and compare.
15 minutes	Who was correct with their guess? Draw the class together and compare results and methods. Ask them to display their results on the flipchart paper and discuss amongst the group What assumptions have been made? How would this be used in the 'real world' and what skills could be used in other tasks

These extensions are optional.
They could well be used as another session to follow on or at a later date

	Content
	Is there a height and weight where the amounts would be equal? How could we find out?
	What do you think the result would be if we used £5 notes for both? This could be done as a planning exercise and searching for the weight and thickness of a £5 note on the internet reveals some interesting maths facts.

How many pictures could I hang on the wall? (Entry 2 – Level 2 depending on support)

Introduction

The aim of this activity is to give learners practice in problem solving using a range of measuring skills and possibly area calculations in a very practical way.
It will also be an opportunity to incorporate assumptions and planning skills.
Being in no particular context it should be applicable to all learners but could be contextualised quite readily if needs be. This also means that once completed the skills and processes practiced within the task can be identified and discussed to see where they might be used in other, more practical situations.

I have used this task with care and sports students and found I was able to justify the relevance by relating it to sports achievement certificates or health and safety certificates.

The numeracy skills in this task could be kept to using one or two significant figures without taking into account the odd shapes of the wall such as light switches or doors and windows and hence the task could possibly be adaptable for entry level learners. Alternatively, you could set some conditions like the pictures must be higher that 1 metre, lower than 2.5 metres and no closer than 30cm to the wall. The gap could be half a picture with or state a measurement.
In the room I use, there is a door on one wall that learners have to work up to. Another wall is clear so I had rules as above, but two groups used it with different orientation of the pictures. A group of level 2 learners tackled the task with the condition that there must be an allowance for approximately 25% of the pictures to be landscape.

Some of the useful functional skills that this can be used to highlight and practice are:

- Making a plan of a task – Representing
- Deciding on levels of accuracy required and affect of rounding on the outcome – Analysis of the data
- Assumptions – eg the numbers don't divide exactly so round up or down and get more pictures in or spread them out more – Interpretation of the answer
- Symmetry from the centre or measure from one wall? Decision making

To make sure you are practising the functionality of the learners' maths skills, the numeracy skills required should be checked before hand to make sure a weakness here is not a barrier to tackling the problem.

FA1 General problem solving skills	
1.1 – Solve problems using mathematical skills	Candidates should be able to: a) select and use suitable problem solving strategies and efficient techniques to solve numerical problems; b) identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches; c) break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods; d) use notation and symbols correctly and consistently within a problem; e) use a range of strategies to create numerical representations of a problem and its solution; move from one form of representation to another in order to get different perspectives on the problem; f) interpret and discuss numerical information presented in a variety of forms; g) present and interpret solutions in the context of the original problem; h) review and justify their choice of mathematical presentation; i) understand the importance of counter-example and identify exceptional cases when solving problems; j) show step-by-step deduction in solving a problem; k) recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying those assumptions may have on the solution to a problem.

FA9 General measures	
9.1 – Interpret scales and use measurements	Candidates should be able to: a) interpret scales on a range of measuring instruments, including those for time and mass; b) know that measurements using real numbers depend on the choice of unit; c) understand angle measure using the associated language; d) make sensible estimates of a range of measures in everyday settings; e) convert measurements from one unit to another; f) know rough metric equivalents of pounds, feet, miles, pints and gallons.

Resources needed

Calculators, range of suitable (and non suitable) measuring equipment, 25cm x 35cm template (or whichever size you choose), squared or graph paper blue-tack maybe useful to position template on wall.

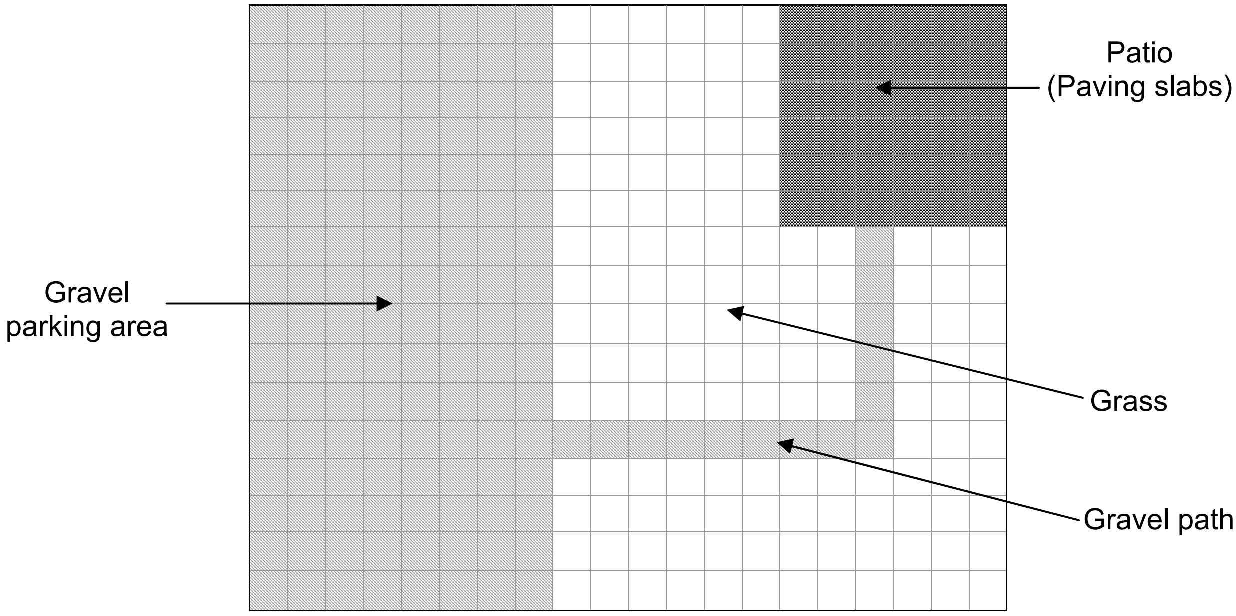
Misconceptions and common errors

Some learners may not see the process of planning the task as important.
Visualisation of spatial problems – this will become clear when planning.
Confusion between switching from units – cm to m.
Reading measuring tool accurately/correctly.

Previously set Functional Maths Question (extract)
January 2009, level 1

Task 1 – Garden Plans

The scale drawing shows George’s design for a new garden.



Scale: 1 cm represents 1 m

(a) What is the length of the longest edge of the gravel path?
Write your answer in metres.

(b) George can buy square paving slabs. Each one has sides 50 cm long.

How many paving slabs does he need to cover the patio?

(c) What is the area of the path?

(d) How much edging board will George need between **all** the gravel and the grass?

The Principal Examiner commented:

Task 1: Garden Plans

Part (a)	Less able candidates struggled to use the scale to determine sensible answers to parts (a) to (d). They also struggled to use appropriate units for length and area. The expected answer was 4.5 m but 7.5 m was also accepted. Answers such as 17 m were not uncommon.
Part (b)	It was anticipated that candidates might find this easy, as they simply had to count squares. Many did just this. However, candidates often reached 36 but then doubled or halved.
Part (c)	Many candidates struggled to find the area of the path. A common error was to regard the path as a single rectangle and to attempt to multiply a length by a width. This could be 9 x 6 but might have been 8 x 5 and many other strange answers were seen. Those who counted the squares (14) frequently halved, mistakenly thinking that, because 2 x 50 cm = 1 metre, then a square that is 50 cm by 50 cm will fit into a 1 metre square twice rather than 4 times.
Part (d)	This question posed many problems to candidates of all abilities. The correct answer was rarely seen.

In general, candidates found difficulty in the following areas:

- Deciding which data was relevant to answering the task
- Deciding what the question was asking them to do
- Deciding the strategy required to answer the question
- Annotating their work
- Using results to support their answers.

Main Activity: How many pictures could I hang on the wall?

You have a collection of drawings and paintings on A4 sheets of paper.

When they are framed they measure 25cm x 35cm

How many could you hang on the wall?

- Before you start make an estimate of how many pictures you could fit on the wall.
- Next you need to develop a plan of how you will go about solving the problem.
- You must write a list of the information you will have to collect before you can start the task.
- Make sure your plan shows the correct order of operation, include any assumptions you have to make and don't forget to write any hints to yourself so you don't forget things as you work through the task.
- When you are happy that your plan is complete, check it through with your tutor / teacher and then you can start the task.
- As you work through, make sure you write the measurements and units carefully and explain any calculations you do so you can describe your methods to someone afterwards.
- Complete the task by drawing a diagram to show how you have decided to hang the pictures to show them off at their best.

When you have finished…

- How close were you to your estimate?
- Did you follow your plan or did you have to adapt it along the way?
- Is there a different way of solving the problem?

Lesson plan: How many pictures could I hang on the wall?

Time	Content
5 - 10 minutes	Review understanding of measurement – confirm learners are comfortable using a range of measuring tools such as rulers, tapes and tape measures. Also that they are able to measure in millimetres, centimetres and metres and convert between the three units. Review understanding of area and measurement – including the importance of using the correct units and level of accuracy required for the task.
10 - 15 minutes	Hand out a few A4 pictures mounted on card measuring 25cm x 35cm. *If you are in a classroom it is a good idea to split the group so that each group has a different wall to work on, or uses the pictures in a different orientation*. *I always suggest that the pictures should be no lower than 1 metre and no higher than 2 metres but this could be changed depending on the situation.* *You could mark out areas on the wall with masking tape to start them off* Introduce the task (see notes) and get an estimate of how many pictures they think may fit on the wall before any maths has been done – record these for comparison at the end. Discuss what information might be needed to complete the task and how this data can be collected and what maths might be needed to complete the task. Decide on what equipment will be needed to obtain the measurements. Discuss and agree a plan of how to complete the task. *This stage may be done as a group discussion or could be done in small groups or pairs with the groups feeding back. This then elicits comparisons, promotes constructive criticism and may highlight a variation in methods and accuracy.* *Depending on the level of the learners, you may need to give guidance on where to start i.e. leave a 30cm gap from the wall and a 20cm gap between each picture.* *For Level 2 learners you may want to introduce a higher level of difficulty by saying the middle row must be portrait while the others should be landscape, or that every third picture should be landscape.* *Measuring a centre line and working outwards is a useful technique as is dividing one picture width plus gap into the wall length.*
25 - 30 minutes	In their groups, the learners can now start to follow their plan, measuring the wall and trying to fit the pictures accurately. There may be opportunity for some to work practically from the start and some to start with a plan. Lower level learners may need to use the templates and use trial and improvement.
10 minutes	Draw the class together and compare results and methods. Who was closest to their estimate? Which display would be the most interesting? What assumptions have been made? How would this be used in the ‘real world’ and what skills could be used in other tasks?

These extensions are optional.
They could well be used as another session to follow on or at a later date

Time	Content
	Only 30% of the wall should be covered with pictures
	Use different sized pictures
	Where should the nails be placed?

Build a Jenga set (Entry 3 – Level 1)

Introduction

This activity introduces area and volume with a game to play at the end. It covers measurement and drawing, nets, 3d objects, tallying and much of it is practical. There is no answer to be found but many decisions need to be made throughout the task and so it practises the functionality of the skills throughout.

It is in no fixed context or vocational area so should be applicable to all learners and present an everyday problem that could be related to many scenarios.

Throughout the task there are many areas that can be explored further, so the whole process can take a couple of sessions or quite reasonably be used to practise skills over four or five sessions. Once completed, or during the task, the skills and processes practiced can be identified and discussed to see where they might be used in other, more practical situations.

I have used this task as an activity with a mixed group of learners from Entry 3 to Level 1 and it has stretched both in different ways. I have used it to teach nets, and confirm measurement. It is useful for justifying accuracy and the importance of following instructions.

The numeracy skills in this task are probably no more than Entry 3 (Level 1 in places if you do the extended task) so the primary learning is the functionality of those skills.

Some of the useful functional skills that this can be used to highlight and practice are:

- Making a plan of a task – Representing
- Deciding on levels of accuracy required when drawing and the affect of rounding on the outcome – Analysis of the data
- How to collect and measure data when collecting the results of the game
- How to show and communicate the results – Interpretation
- Interpretation of results specifically from the extension task.

To make sure you are practising the functionality of the learners' maths skills, the numeracy skills required should be checked before hand to make sure a weakness here is not a barrier to tackling the problem.

Key stage 4 references:

FA1 General problem solving skills	
1.1 – Solve problems using mathematical skills	Candidates should be able to: a) select and use suitable problem solving strategies and efficient techniques to solve numerical problems; b) identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches; c) break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods; d) use notation and symbols correctly and consistently within a problem; e) use a range of strategies to create numerical representations of a problem and its solution; move from one form of representation to another in order to get different perspectives on the problem; f) interpret and discuss numerical information presented in a variety of forms; g) present and interpret solutions in the context of the original problem; h) review and justify their choice of mathematical presentation; i) understand the importance of counter-example and identify exceptional cases when solving problems; j) show step-by-step deduction in solving a problem; k) recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying those assumptions may have on the solution to a problem.

FA9 General measures	
9.1 – Interpret scales and use measurements	Candidates should be able to: a) interpret scales on a range of measuring instruments, including those for time and mass; b) know that measurements using real numbers depend on the choice of unit; c) understand angle measure using the associated language; d) make sensible estimates of a range of measures in everyday settings; e) convert measurements from one unit to another; f) know rough metric equivalents of pounds, feet, miles, pints and gallons.

Resources needed

Card, rulers, Scissors or knives, pencils and rubbers, glue, colouring pens/pencils.

Misconceptions and common errors

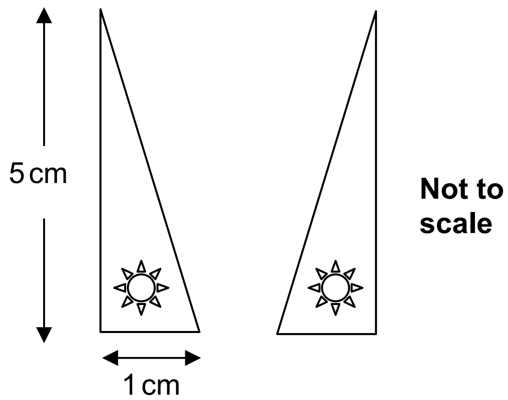
Some learners may have difficulty cutting/sticking.
Drawing lines inaccurately leading to cuboid not fitting neatly.
Some learners unable to visualise purpose of tabs or where they should go.
With the extension task – trying to visualise the mathematical answers to a problem.

Previously set Functional Maths Question (extract) November 2009

Task 3 – Jewellery

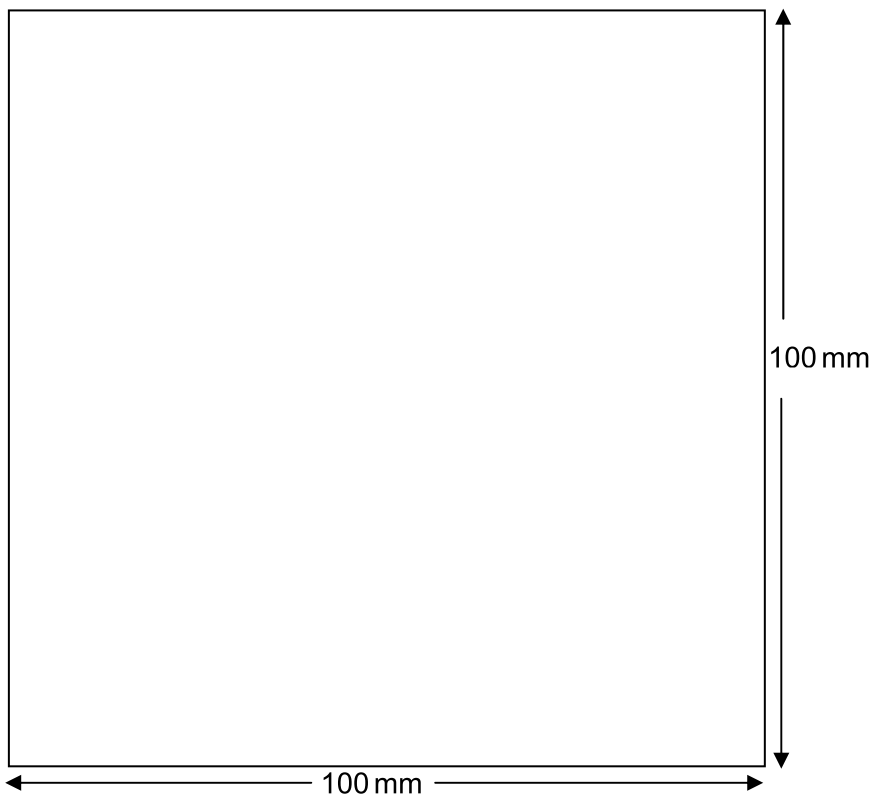
Amy has a flat sheet of silver, in the shape of a square. Each side is 100 mm long.

Amy designs and makes pairs of silver earrings.
One of her designs is shown. Each earring is a right-angled triangle with a pattern on it.



Amy thinks that she can cut 20 pairs of these earrings from her sheet of silver.

(a) Use this square to help explain whether Amy is correct.



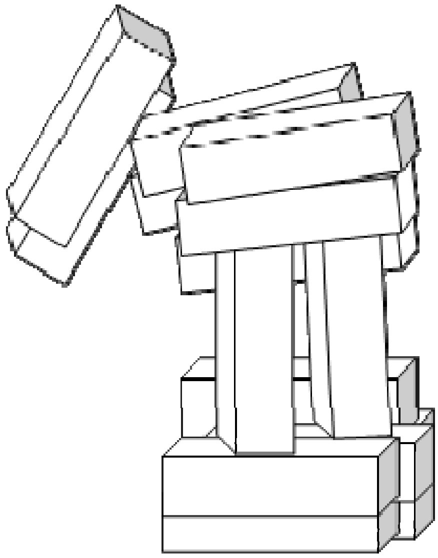
The Principal Examiner commented:

Candidates needed to show that a rectangle 1cm by 5cm would be sufficient to make one PAIR of earrings; then to draw sufficient of these on the square (all, a row or quarter); then complete with a comment. Where broad pencil use showed that 20 pairs could not be made, this, with an appropriate comment was awarded full marks. Candidates who used area scored a maximum of 4 marks. Less able candidates drew random rectangles or triangles of indeterminate and varying size and used their drawings either to agree or disagree with Amy. They did not score well.

Main Activity: Build a Jenga Set

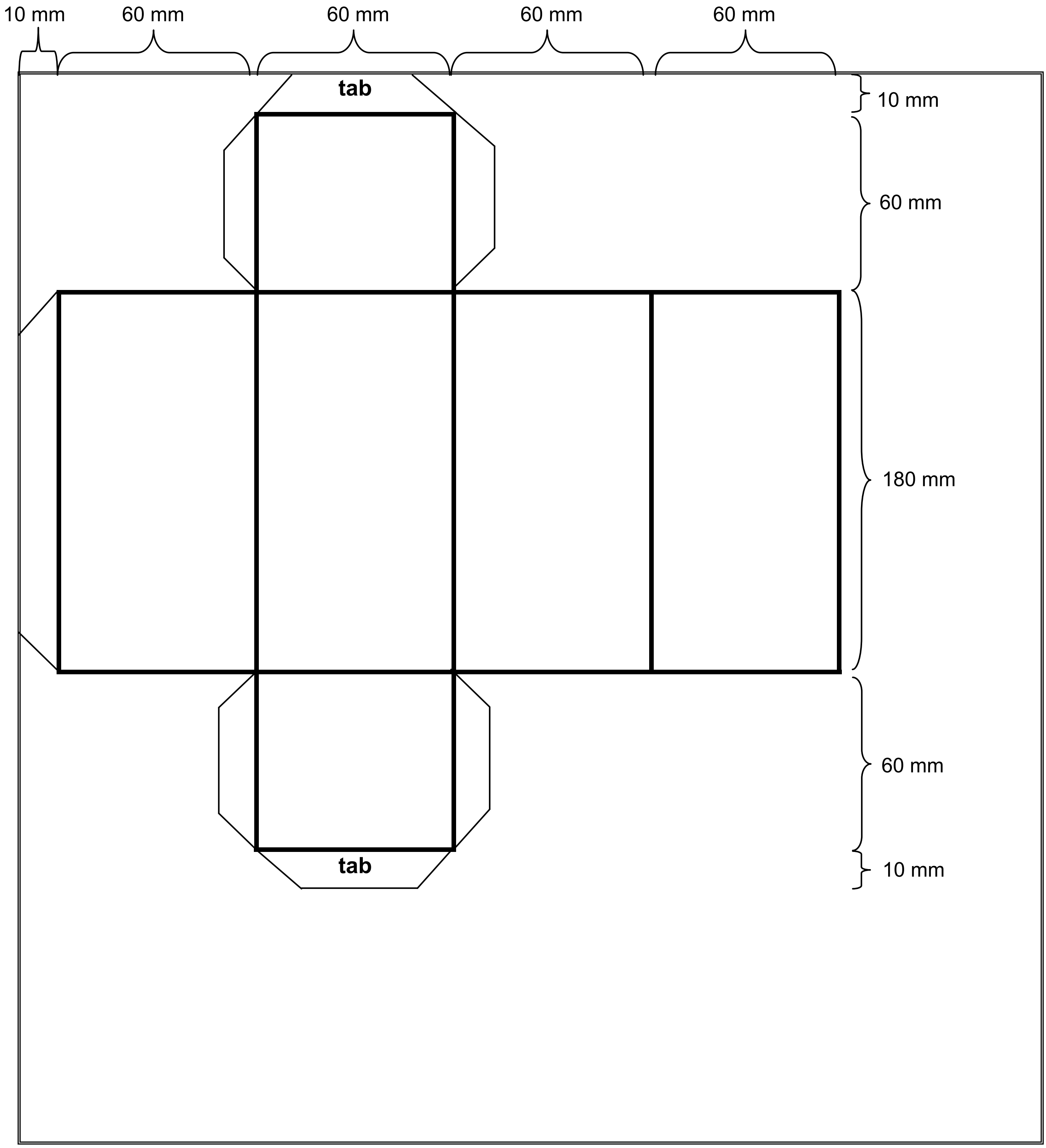
The task here is to each make a Jenga block and complete the challenge at the end.

- You need to develop a plan of how you will go about the task.
- You must write a list of the materials you will need before you can start the task.
- Make sure your plan shows the correct order of operation, includes any assumptions you have to make and don't forget to write any hints to yourself so you don't forget things as you work through the task.



- When you are happy that your plan is complete, check it through with your tutor / teacher and then you can start the task.
- Build your block and decorate it if you wish.
- When everyone has completed their block the final challenge can begin.
- You have 30 seconds to build a tower of blocks as high as you can. When 30 seconds is up, whatever height the tower is then, is what counts.
- You must decide how to record, how to check and how to display the results.

Diagram of net for Jenga cube



Lesson plan: Build a Jenga set

Time	Content
5 - 10 minutes	Review 3d shapes, properties and nets – confirm learners are familiar with nets and how to construct 3d objects using nets. Also that they are able to measure in millimetres and centimetres and convert between the units.
10 - 15 minutes	Introduce the task: building a Jenga set and suggesting there is a game of skill at the end. Discuss how we can go about constructing a cuboid with the right dimensions to fit together as a Jenga set does i.e. the length must be 3 x the square cross section. Decide on what equipment will be needed. Discuss and agree a plan of how to complete task. At this stage I show an example of how best to draw the net on a rectangular piece of card. I suggest the dimensions of the finished cuboid should be 60mm x 60mm x 180mm. (ratio of 3:1:1). A good exercise is to ask the learners to decide where to put the tabs to show their understanding of how the net will form the shape. The tabs for sticking should be 10mm (or I have used 10% for some learners) and ask group to decide, on a rough sketch, where the tabs need to be (see diagram 1) This helps them to picture the net and how it will form the final shape. It may be useful to have a cut out and scored net, with tabs, ready made so you can show it as an example at some point.
20 - 30 minutes	Learners now draw, cut out and stick their nets to make the cuboids. This often takes learners a wide variation in time with some needing more support or guidance so suggesting that learners decorate or colour their 'boxes' often levels the group to the same stage. Alternatively, asking the stronger to support the weaker ones can be very conducive to good team working.
20 minutes (depending upon numbers)	The game – each member of the group has 30 seconds to build a tower as high as possible using as many of the 'blocks' as possible. How ever tall their tower is when 30 seconds is up is what counts so if it falls after 25 seconds and they start again, that is what counts. Ask the group how you could measure the height of the tower and record the results. Suggest they call a block on it's side 'S' and a block end up 'E' S will equal 60mm and E will equal 180mm Collect the scores for each as a tally (example) and then calculate height at end of game. To display the results they could be put into a bar chart as a good comparison.

This exercise is optional.
It could well be used as another session to follow on

Time	Content
60 minutes	How many Jenga blocks fit into a range of different sized boxes I use this as a follow up session later to show the practical side to the problem and then the mathematical reasoning, I use the ‘one volume divided by the other volume to get a value and then ask the learners to try and make the blocks fit in the box – obviously it won’t work even though it is mathematically correct. I then get the learners to do it by dividing the block length into the box length etc. This is a really nice example of interpretation of mathematical results.

How many Maltesers could I fit in this room? (Entry 2 – Level 2)

Introduction

The aim of this activity is to give learners practice in problem solving using a range of measuring skills and volume calculations in a very practical way.

It will also be an opportunity to incorporate assumptions and planning skills.

Being in no particular context it should be applicable to all learners but could be contextualised quite readily if needs be. This also means that once completed the skills and processes practiced within the task can be identified and discussed to see where they might be used in other, more practical situations.

I have used this task as an activity after I have taught measurement using metric units of length, but often before I have introduced the concept of area and volume.

The numeracy skills in this task could be kept to using one or two significant figures without taking into account the odd shapes within the room and hence the task could possibly be adaptable for entry level learners. Alternatively, you could set some conditions like level of accuracy required, spaces that must be included, average size of Maltesers which could increase the level of the task. In the room I used part of the ceiling was sloping towards the windows so the volume of the room had to be found in sections using three sections, halving one of them. If you think imaginatively, this could easily become quite an involved investigation.

Some of the useful functional skills that this can be used to highlight and practice are:

- Making a plan of a task – Representing
- Deciding on levels of accuracy required and affect of rounding on the outcome – Analysis of the data
- Assumptions – eg could you pile Maltesers 250 high without the bottom ones collapsing? – Interpretation of the answer
- Would they pile exactly on top of each other or would they stack? What sort of pattern would they fall into?

To make sure you are practising the functionality of the learners' maths skills, the numeracy skills required should be checked before hand to make sure a weakness here is not a barrier to tackling the problem.

Key stage 4 references:

FA1 General problem solving skills	
1.1 – Solve problems using mathematical skills	Candidates should be able to: a) select and use suitable problem solving strategies and efficient techniques to solve numerical problems; b) identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches; c) break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods; d) use notation and symbols correctly and consistently within a problem; e) use a range of strategies to create numerical representations of a problem and its solution; move from one form of representation to another in order to get different perspectives on the problem; f) interpret and discuss numerical information presented in a variety of forms; g) present and interpret solutions in the context of the original problem; h) review and justify their choice of mathematical presentation; i) understand the importance of counter-example and identify exceptional cases when solving problems; j) show step-by-step deduction in solving a problem; k) recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying those assumptions may have on the solution to a problem.

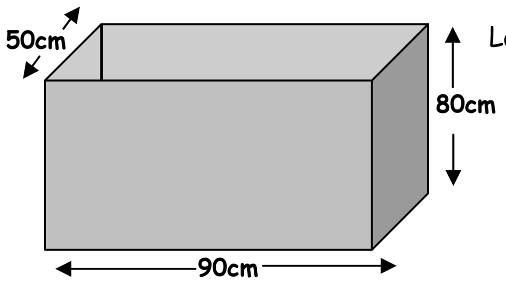
FA9 General measures	
9.1 – Interpret scales and use measurements	Candidates should be able to: a) interpret scales on a range of measuring instruments, including those for time and mass; b) know that measurements using real numbers depend on the choice of unit; c) understand angle measure using the associated language; d) make sensible estimates of a range of measures in everyday settings; e) convert measurements from one unit to another; f) know rough metric equivalents of pounds, feet, miles, pints and gallons.

Resources needed

Calculators, range of suitable (and non suitable) measuring equipment, pack of Maltesers between groups / pairs.

Misconceptions and common errors

Weaker learners may not see the process of planning the task as important.
Visualisation of very large amounts – this will become clear when estimating amounts.
Confusion between switching from units.
Reading measuring tool accurately/correctly.



For Sale

Large aquarium – almost new £80 ono

Owner moving so quick sale.

Buyer must collect

Tel: 01310 323252

(b) Before buying the aquarium, Liam wants to work out how many snake fish it can hold. If there are too many, they may fight or catch diseases.

Looking in some books, he finds two rules:

HOW TO FIND THE SAFE LENGTH OF FISH TO KEEP IN AN AQUARIUM

Surface area rule
1 cm length of fish for every 12 cm^2 of aquarium water surface area

Volume rule
1 cm length of fish for every 1.8 litres of aquarium water

Liam is cautious and decides to use the rule that gives the smaller safe total length of fish for this particular aquarium. Show which rule he should choose.

The Principal Examiner commented:

Part (b)

A problem that a significant number of candidates had with this section is that they incorrectly considered the surface area of the water to be the sum of the areas of all sides, including top and bottom. Those that avoided this mistake tended to have little trouble with the rules for stocking fish. At times, in this section and other sections, a number of candidates had difficulty with **units of area and volume**. **There was some confusion with the units for volume and area and the fundamental difference between these two measures**.

Main Activity: How many Maltesers could I fit in this room?

You have a packet of Maltesers between you. Your task is to try and calculate how many would fit in the classroom.

- Before you start make an estimate of how many you think might fit the room.
- Next you need to develop a plan of how you will go about solving the problem.
- You must write a list of the information you will have to collect before you can start the task.
- Make sure your plan shows the correct order of operation, includes any assumptions you have to make and don't forget to write any hints to yourself so you don't forget things as you work through the task.
- When you are happy that your plan is complete, check it through with your tutor / teacher and then you can start the task.
- As you work through, make sure you write the measurements and units carefully and explain any calculations you do so you can describe your methods to someone afterwards.

When you have finished…

- How close were you to your estimate?
- How accurate do you think you are? To the nearest 10, 100, 1000?
- Is there a different way of solving the problem?

Lesson plan: How many Maltesers could I fit in this room?

Time	Content
5 - 10 minutes	Review understanding of measurement – confirm learners are comfortable using a range of measuring tools such as rulers, tapes and tape measures. Also that they are able to measure in millimetres, centimetres and metres and convert between the three units. Review understanding of area and volume – including the importance of using like units.
10 - 15 minutes	Hand out a packet of Maltesers – one packet per group/pair. Introduce the task (see notes) and get an estimate of how many Maltesers may fit in the room before any maths has been done – record these for comparison at the end. Discuss what information might be needed to complete the task and how this data can be collected. Decide on what equipment will be needed to obtain the measurements. Discuss and agree a plan of how to complete task. This stage may be done as a group discussion or could be done in small groups or pairs with the groups feeding back. This then elicits comparisons, promotes constructive criticism and may highlight a variation in methods and accuracy.
20 - 25 minutes	In small groups learners should now be able to collect the data they need. Measure the length, width and height of the room using measuring tools provided and record data. Following the plan, the mathematics can be done to find out how many Maltesers fit in the room.
10 minutes	Draw the class together and compare results and methods. Who was closest to their estimate? Whose answer is most accurate? What assumptions have been made?

This extension is optional.
It could well be used as another session to follow on

Time	Content
10 - 15 minutes	How accurate is and could the answer be? Is the room a perfect cuboid? Is there more space for window cavities etc?
25–30 minutes	How long would it take to eat all the Maltesers in the room? How much would it cost to buy all the Maltesers in the room?
10 minutes	Where might we ever need to do an exercise like this at work or home? i.e.: Why could knowing these skills be useful?
15 minutes	Could you set a problem like this for your peers?

RECOGNISING ACHIEVEMENT

Functional Skills Maths

Functional skills qualification in Maths at Level 1 and Level 2

Statistics

Interpreting lists (Sandwiches) (Entry 2 and 3) (Also Level 1)

Introduction

The aim of the activity is to give learners practise in extracting information from tables and using it to solve simple questions. It also works on the general area of arithmetic.

The task could be introduced to the whole group and issues discussed.

- What information is in the table?
- How is this different to the starter?
- How may sales differ in Sean’s area to Layla’s?

Some questions have been posed, but these are not set in stone. The intention is to provide a platform for using the data. **Not all the questions need to be used. Others may be substituted. If learners are not very independent** then only one or two questions may be used. Eg

The solutions to these questions could be calculated in pairs and the answers discussed. Further questions may then be “fed into” the mix and again worked on and discussed.
Learners may then be asked for ways that the task might be developed.
Further questions could be:

- How much money could Sean make in a week?
- How much profit does Layla make in a week?
- What other factors could affect Layla’s profit?
- Is all the money she makes profit?
- What is a reasonable amount (hourly) to work for?

These alternatives, and there are many others, would involve learners extracting information and answering a task.

The task is ideal for ICT usage and any spreadsheet could be used to explore changes in prices. Learners could estimate the cost of making sandwiches for themselves. They could engage in research around their school or college, at the canteen etc.

The table may be simplified. Rows may be taken out or simpler numbers used.

Sandwich	Cost
Cheese	£0.60
Ham and Cheese	£0.80
Cream Cheese and Pineapple	£1.30
Tuna Mayo	£0.65
Bacon	£1.50
Egg Mayo	£0.40

No simplification required for E3 (or Level 1)

Learners could work in small groups to solve problems and present results.

It is important that outcomes are discussed and learners justify their assertions with use of data.

Spreadsheets could be prepared for learners to use and their own data could be inserted.

Functional Standards

E2		E3	
R1	Extract **relevant** information from the tables	R1	Extract **relevant** information from the tables
R2	Decide to add or subtract to answer questions such as 1 and 3	R2	Decide, for example, to calculate profit from each sandwich and multiply by sales of each in response to question 3
		R3	Add or subtract to answer questions such as 1 and 3
A1	Add or subtract to answer questions such as 1 and 3 Follow result of discussion, for example, to calculate profit from each sandwich and multiply by sales of each in response to question 3	A1	Follow own strategy, for example, to calculate profit from each sandwich and multiply by sales of each in response to question 3
A2	Record results correctly		
A3	Respond positively when asked to repeat calculations or check that results seem sensible, or not.	A2	Repeat calculations or note that results seem sensible, or not.
I1	Explain answers and refer to results	I1	Explain answers and refer to results

Resources Needed

Calculators, rulers and pencils

Misconceptions and common errors

Extracting information from tables.

- Candidates frequently fail to take note of headings in tables.
- They transcribe data from the tables incorrectly.
- Candidates often do not show the working they have used to obtain answers and so it is often difficult to know whether they have used correct figures or not.
- Answers are often poorly expressed. Learners often write at length but fail to communicate ideas clearly. Simple notes and headings are often preferable to prose though, at E2 and E3, answers may be given verbally. The point still, however, applies.

Previously set Functional Mathematics questions
Level 1 March 09

The Royal Mail charges different postage rates depending on the measurements of each item to be posted.

Type of letter	Length mm	Width mm	Thickness mm
Letter	240	165	5
Large Letter	353	250	25

Each measurement **must not** be bigger than the figure given.

(a) Which of these post cards must be posted as a large letter?

A

15 cm

23 cm

B

90 mm

250 mm

The Principal Examiner commented:

1a Most candidates answered correctly, although some provided evidence that conflicted with their answer such as incorrect conversions of cm to mm or vice versa. In such cases, which were rare, a mark was lost.

Paolo looks up this information on temperatures in the two cities.

Average monthly temperatures in °C

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rome	7.2	8.3	10.5	13.7	17.8	21.7	24.4	24.1	20.9	16.6	11.7	8.4
New Delhi	14.1	16.9	22.4	28.6	32.9	33.8	31	29.8	29.2	26	20.3	15.4

(A graph was also presented)

(b) Use the information to describe the average monthly temperatures in the two cities.

The Principal Examiner commented:

Part (b) Some good answers were seen in which trends were identified and described. The expected solutions showing the seasonal rise and fall, peaking in different months, June (New Delhi) and July (Rome), and New Delhi always being the hotter city were often seen although not always together.
Candidates need to know that, as this was a six-mark question, three components were expected to the solution. These were for:

- identifying certain temperatures relevant to the response
- describing the trend
- quantifying the differences between the cities (Using the temperatures).

Common errors were to:

- describe the trends but not use any specific temperatures
- write generally but not attribute comments to a specific city
- re-list the temperatures but make no further comment.

Less able candidates misinterpreted the question and thought that an estimate of the average was required. “It’s June” and “Rome about 20 and New Delhi about 25” were often seen.

Starter Activity: Interpreting lists (Sandwiches)

This is intended as a group activity.	
Learners could suggest sandwiches they would choose that are not on the list and could vote for the popularity of the types of sandwich.	R1, A1, A2, I1
Data could be collected in advance, at the same time or later to add to the "votes" of the group.	A2
If this is to be used solely as a starter to the lesson, advance collection (by asking colleagues teaching other groups?) and adding this, live, to the results is helpful. This produces a larger sample and the merits of this may be discussed.	A2
Learners could also suggest price ranges they would be prepared to pay and think reasonable.	R1, A1, A2 (E2), I1
Data on prices could be collected as an "out of hours" activity from local sandwich bars and these compared with learners own choices.	R1, R2, A1, A2 (E2), I1
An "average" price could be found.	R1, R2, A1, A2, I1
At stages in the work learners should be encouraged to check that results are consistent and contain no detectable errors.	A3 (E2) or A2 (E2)

A blank list

Sandwich	Tally	Frequency
Cheese		
Ham and Cheese		
Cream Cheese and Pineapple		
Tuna Mayo		
Bacon		
Egg Mayo		

Starter Activity: Interpreting lists (Sandwiches)

Sean wants to open a “sandwich bar”.
His friend, Layla, sells these sandwiches in Barton, a town in the next county.

Cheese
Ham and Cheese
Cream Cheese and Pineapple
Tuna Mayo
Bacon
Egg Mayo

1 Would these be good sandwiches for Sean to sell?
2 Would people in your group buy them?

Main Activity: Interpreting lists (Sandwiches)

Sean wants to open a “sandwich bar”.
His friend, Layla, sells sandwiches in Barton, a town in the next county.
Layla gives Sean this information about the sandwiches she sells.

Sandwich	Price	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Cheese	£2.50	6	4	2	5	6	10	14
Ham and Cheese	£2.80	8	6	5	9	10	15	19
Cream Cheese and Pineapple	£3.20	3	2	2	3	4	5	5
Tuna Mayo	£2.90	10	8	6	9	12	16	24
Bacon	£3.60	12	8	5	11	15	21	32
Egg Mayo	£2.70	6	10	8	6	14	16	18

Salad garnish add £0.60 Portion of chips £1.80

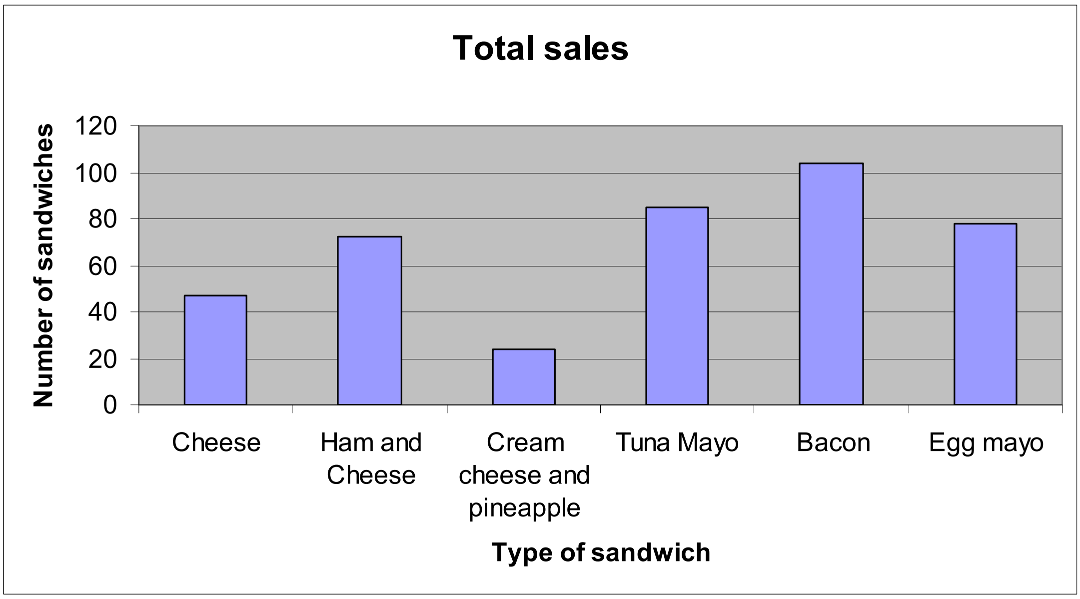
This is what the ingredients cost Layla for each sandwich.

Sandwich	Cost
Cheese	£0.65
Ham and Cheese	£0.84
Cream Cheese and Pineapple	£1.34
Tuna Mayo	£0.65
Bacon	£1.46
Egg Mayo	£0.36

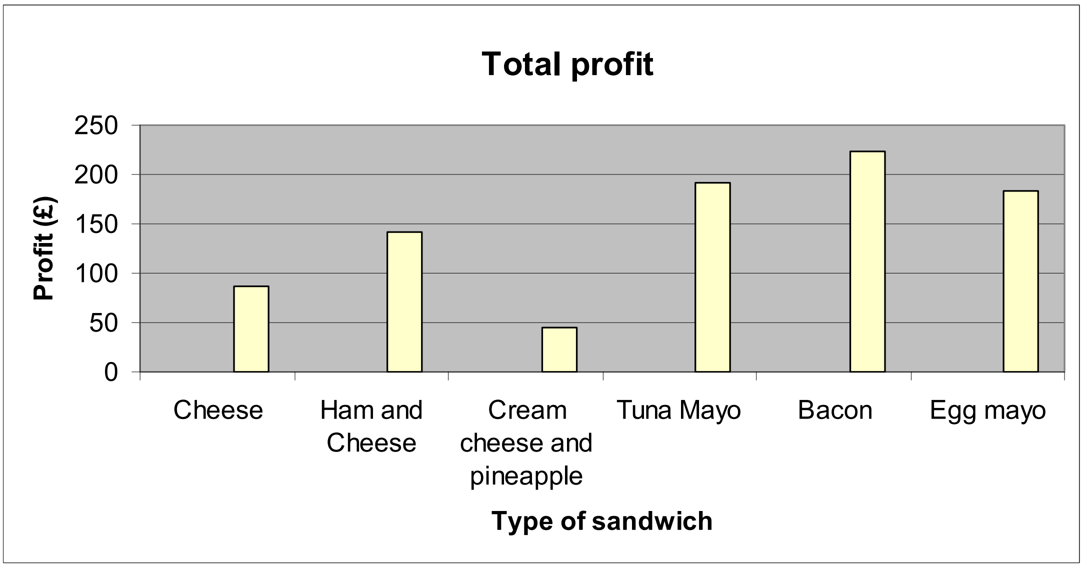
1 How many of each type of sandwich does Layla sell?
2 How much profit does Layla make on one cheese sandwich?
3 How much profit does she make on Monday?
4 Sean wants to open the sandwich bar for six days a week. Which day should he close?
5 Why did you choose this day?
6 What sandwiches should Sean make?
7 Why did you choose these sandwiches?
8 How much money could Sean make in a week?

Sandwich	Price	Cost	Profit	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Total
Cheese	£2.50	£0.65	£1.85	6	4	2	5	6	10	14	47
Ham and Cheese	£2.80	£0.84	£1.96	8	6	5	9	10	15	19	72
Cream Cheese and Pineapple	£3.20	£1.34	£1.86	3	2	2	3	4	5	5	24
Tuna Mayo	£2.90	£0.65	£2.25	10	8	6	9	12	16	24	85
Bacon	£3.60	£1.46	£2.14	12	8	5	11	15	21	32	104
Egg Mayo	£2.70	£0.36	£2.34	6	10	8	6	14	16	18	78

Sandwich	Profit	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Total
Cheese		£11.10	£7.40	£3.70	£9.25	£11.10	£18.50	£25.90	£86.95
Ham and Cheese		£15.68	£11.76	£9.80	£17.64	£19.60	£29.40	£37.24	£141.12
Cream Cheese and Pineapple		£5.58	£3.72	£3.72	£5.58	£7.44	£9.30	£9.30	£44.64
Tuna Mayo		£22.50	£18.00	£13.50	£20.25	£27.00	£36.00	£54.00	£191.25
Bacon		£25.68	£17.12	£10.70	£23.54	£32.10	£44.94	£68.48	£222.56
Egg Mayo		£14.04	£23.40	£18.72	£14.04	£32.76	£37.44	£42.12	£182.52
	Total	£94.58	£81.40	£60.14	£90.30	£130.00	£175.58	£237.04	£869.04



Total sales
Number of sandwiches
120
100
80
60
40
20
0
Cheese
Ham and Cheese
Cream cheese and pineapple
Tuna Mayo
Bacon
Egg mayo
Type of sandwich



Total profit
Profit (£)
250
200
150
100
50
0
Cheese
Ham and Cheese
Cream cheese and pineapple
Tuna Mayo
Bacon
Egg mayo
Type of sandwich

Interpreting lists (Tennis) (Entry 2 and 3, also Level 1)

Introduction

The aim of the activity is to give learners practise in extracting information from tables and using it to solve simple questions.

The task could be introduced to the whole group and issues discussed:

- What information is in the table?
- The meaning of abbreviations.

Some questions have been posed, but these are not set in stone. The intention is to provide a platform for using the data. **Not all the questions need be used. Questions may be replaced or tailored for the group. Greater independence may be given to learners by introducing more open questions.**

- Are all countries equally successful at producing good tennis players?
- If players from each country were joined to form a team (such as Davis Cup), how many points would each country have? OR
- What order would the countries be placed in?

These alternatives, and there are many others, would involve learners extracting information and answering a task.

Information has been presented in the way it is seen on the ATP rankings but the table may be simplified. Rows may be taken out or simpler numbers used.

E2

			Tournaments Played
1	Federer, Roger (SUI)	10 000	19
2	Nadal, Rafael (ESP)	6 900	17
3	Djokovic, Novak (SRB)	6 400	21
4	Murray, Andy (GBR)	5 600	17
5	Davydenko, Nikolay (RUS)	5 100	24
6	Del Potro, Juan Martin (ARG)	5 100	17
7	Soderling, Robin (SWE)	4 800	25
8	Roddick, Andy (USA)	4 600	20
9	Verdasco, Fernando (ESP)	3 600	27
10	Tsonga, Jo-Wilfried (FRA)	3 200	24
11	Ferrer, David (ESP)	3 000	25
12	Cilic, Marin (CRO)	2 900	23
13	Gonzalez, Fernando (CHI)	2 400	21
14	Youzhny, Mikhail (RUS)	2 400	27
15	Monfils, Gael (FRA)	2 200	25
16	Ljubicic, Ivan (CRO)	2 100	25
17	Berdych, Tomas (CZE)	2 100	27
18	Ferrero, Juan Carlos (ESP)	2 100	25
19	Isner, John (USA)	1 900	25
20	Stepanek, Radek (CZE)	1 700	21

E3

			Tournaments Played
1	Federer, Roger (SUI)	10 000	19
2	Nadal, Rafael (ESP)	6 880	17
3	Djokovic, Novak (SRB)	6 410	21
4	Murray, Andy (GBR)	5 570	17
5	Davydenko, Nikolay (RUS)	5 150	24
6	Del Potro, Juan Martin (ARG)	5 120	17
7	Soderling, Robin (SWE)	4 760	25
8	Roddick, Andy (USA)	4 600	20
9	Verdasco, Fernando (ESP)	3 650	27
10	Tsonga, Jo-Wilfried (FRA)	3 190	24
11	Ferrer, David (ESP)	3 010	25
12	Cilic, Marin (CRO)	2 950	23
13	Gonzalez, Fernando (CHI)	2 390	21
14	Youzhny, Mikhail (RUS)	2 380	27
15	Monfils, Gael (FRA)	2 220	25
16	Ljubicic, Ivan (CRO)	2 140	25
17	Berdych, Tomas (CZE)	2 120	27
18	Ferrero, Juan Carlos (ESP)	2 050	25
19	Isner, John (USA)	1 880	25
20	Stepanek, Radek (CZE)	1 710	21

At E3 representing the number of points as a graph would extend the task. The graph could be used to show the relative disparity of the players and learners could explain this.
Learners could work in small groups to solve problems and present results.

It is important that outcomes are discussed and learners justify their assertions with use of data.

Functional Standards

E2		E3	
R1	Extract **relevant** information from the tables	R1	Extract **relevant** information from the tables
R2	Decide to add or subtract to answer questions such as 3 and 5	R2	Decide to subtract and add 1. Consider Federer's results in response to question 4.
		R3	Add or subtract to answer questions such as 3 and 5
A1	Add or subtract to answer questions such as 3 and 5 Follow result of discussion to subtract and add 1. Consider Federer's results in response to question 4.	A1	Follow own strategy to, for example, subtract and add 1 and consider Federer's results in response to question 4.
A2	Record results correctly		
A3	Respond positively when asked to repeat calculations or check that results seem sensible, or not.	A2	Repeat calculations or note that results seem sensible, or not.
I1	Explain answers and refer to results	I1	Explain answers and refer to results

Key Stage 4 references (J562)

FA13	**General data handling**
13.1 - Understand and use statistical problem solving process/handling data cycle	Candidates should be able to: a) carry out each of the **four** aspects of the handling data cycle to solve problems: i) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources; ii) process and represent the data: turn the raw data into usable information that gives insight into the problem; b) Interpret and discuss the data: answer the initial question by drawing conclusions from the data.
13.2 - Experimenting	Candidates should be able to: a) discuss how data relate to a problem, identify possible sources of bias and plan to minimise it; b) identify key questions that can be addressed by statistical methods; c) design an experiment or survey and decide what primary and secondary data to use; d) design and use data-collection sheets for grouped discrete and continuous data; e) gather data from secondary sources, including printed tables and lists from ICT-based sources;

13.3 - Processing	Candidates should be able to: a) draw and produce pie charts for categorical data, and diagrams for continuous data, frequency diagrams (bar charts, frequency polygons and fixed interval histograms) and stem and leaf diagrams; b) calculate mean, range and median of small data sets with discrete then continuous data;
13.4 - Interpreting	Candidates should be able to: a) look at data to find patterns and exceptions; b) interpret a wide range of graphs and diagrams and draw conclusions; c) interpret social statistics including index numbers; and survey data; d) compare distributions and make inferences, using the shapes of distributions and measures of average and range; e) understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better population characteristics.

Resources Needed

Graph paper, calculators, rulers and pencils

Misconceptions and common errors

Extracting information from tables.

- Candidates frequently fail to take note of headings in tables.
- They transcribe data from the tables incorrectly.
- Candidates often do not show the working they have used to obtain answers and so it is often difficult to know whether they have used correct figures or not.
- Answers are often poorly expressed. Learners often write at length but fail to communicate ideas clearly. Simple notes and headings are often preferable to prose though, at E2 and E3, answers may be given verbally. The point still, however, applies.

Graph drawing.

Weaker learners often fail to plan scales.

Common errors are:

- To fail to have equal width columns or equal horizontal spaces between plots
- To create difficult to use continuous vertical scales
- To plot frequencies at equally spaced points on the vertical axis, regardless of the magnitude of each frequency. Thus 200, 500, 520, 810 may each be 2cm apart and simply written against the height of the column on the vertical scale.

Previously set Functional Mathematics questions
Level 1 March 09

See example provided on page 48.

Level 1 June 09

See example provided on page 49.

Starter Activity: Interpreting Lists (Tennis)

The first list shows the ATP top 5 Men's tennis players.
It also shows the country they come from (SUI = Switzerland) the points they have earned and the number of tournaments they have played.

The results were from 4th June 2010.

			Tournaments Played
1	Federer, Roger (SUI)	10,030	19
2	Nadal, Rafael (ESP)	6,880	17
3	Djokovic, Novak (SRB)	6,405	21
4	Murray, Andy (GBR)	5,565	17
5	Davydenko, Nikolay (RUS)	5,145	24

Abbreviation	Country
ESP	Spain
GBR	Great Britain
RUS	Russia
SRB	Serbia
SUI	Switzerland

1 How many points has Federer been awarded?

2 What country does Nadal come from?

3 How many tournaments has Murray played in?

4 Which players have each played 17 tournaments in the 2010 season?

5 How many more points has Federer than Nadal?

Main Activity: Interpreting Lists (Tennis)

The first list shows the ATP top 20 Men's tennis players.
It also shows the country they come from (SUI = Switzerland) the points they have earned and the number of tournaments they have played.

The results were from 4th June 2010.

			Tournaments Played
1	Federer, Roger (SUI)	10 030	19
2	Nadal, Rafael (ESP)	6 880	17
3	Djokovic, Novak (SRB)	6 405	21
4	Murray, Andy (GBR)	5 565	17
5	Davydenko, Nikolay (RUS)	5 145	24
6	Del Potro, Juan Martin (ARG)	5 115	17
7	Soderling, Robin (SWE)	4 755	25
8	Roddick, Andy (USA)	4 600	20
9	Verdasco, Fernando (ESP)	3 645	27
10	Tsonga, Jo-Wilfried (FRA)	3 185	24
11	Ferrer, David (ESP)	3 010	25
12	Cilic, Marin (CRO)	2 945	23
13	Gonzalez, Fernando (CHI)	2 385	21
14	Youzhny, Mikhail (RUS)	2 375	27
15	Monfils, Gael (FRA)	2 220	25
16	Ljubicic, Ivan (CRO)	2 140	25
17	Berdych, Tomas (CZE)	2 115	27
18	Ferrero, Juan Carlos (ESP)	2 050	25
19	Isner, John (USA)	1 880	25
20	Stepanek, Radek (CZE)	1 705	21

Abbreviation	Country
ARG	Argentina
CHI	Chile
CRO	Croatia
CZE	Czechoslovakia
ESP	Spain
FRA	France
GBR	Great Britain
RUS	Russia
SRB	Serbia
SUI	Switzerland
SWE	Sweden
USA	United States of America

1 Which country produced most tennis player in the top 20 rankings?

2 Which country did Ferrero come from?

3 How many points will Andy Murray have to win to overtake Federer?

4 Why might Murray not top the rankings even if he gained 4500 points?

5 The player in 21st place (Nicholas Almagro) was only 15 points behind Stepanek. How many points did Almagro have?

6 Why might Del Potro be Murray's nearest rival?

Notes: Tennis (Interpreting Lists)

This is a short starter that allows understanding of the structure of the table and the information that it contains.

The starter should be undertaken individually, after a chance to ask any questions about the information in the table. Other questions could be used, such as, “How many points will Djokovic need to overtake Nadal?”

Looking up results for women players could extend the task.

<u>Starter activity answers:</u>

1 10 030

2 Spain

3 17

4 Murray and Nadal

5 3150

<u>Main activity answers:</u>

Numbers in the table could be simplified for use at E2 or E3. This is shown in the tables below. The questions remain the same.

E2

			Tournaments Played
1	Federer, Roger (SUI)	10 000	19
2	Nadal, Rafael (ESP)	6 900	17
3	Djokovic, Novak (SRB)	6 400	21
4	Murray, Andy (GBR)	5 600	17
5	Davydenko, Nikolay (RUS)	5 100	24

Answers:

1 10 000

2 Spain

3 17

4 Murray and Nadal

5 3100

E3

			Tournaments Played
1	Federer, Roger (SUI)	10 030	19
2	Nadal, Rafael (ESP)	6 880	17
3	Djokovic, Novak (SRB)	6 410	21
4	Murray, Andy (GBR)	5 570	17
5	Davydenko, Nikolay (RUS)	5 150	24

Answers:

1 10 030

2 Spain

3 17

4 Murray and Nadal

6 3150

Tallying and Graphs (Time Series) (Level 1 and 2)

Introduction

The aim of the activity is to give learners practise in drawing graphs to represent real data and draw conclusions. The purpose of the graph is to see the trends in the popularity of different names given to girls since 1998. This should be used to decide on the numbers of each nameplate that could be made.

The task could be introduced to the whole group and issues discussed.

- What sort of graph is most appropriate to display the information?
- Should all the names be included on the graph? (Issues of time to plot, etc)
- Are there any names that do not appear in all years? (Jade, for instance)
- What scale should be used for the frequency axis of the graph?
- What accuracy could be plotted?
- The purpose of the graph.
- Appropriate titles.
- Can ICT be used?

After drawing the graph learners might work in pairs to decide

i) Which names to recommend and
ii) How many of each nameplate should be made.

Learners should then write **clear** conclusions, not in flowing prose but as brief notes with justifications for proportions from the data. These should be presented to the group and decisions challenged.

Learners may want to consider other issues such as:

- Is the pattern of name popularity reflected in previous years?
- Use the Internet to find data going back to 1940 etc.
- What about boy's names?

And, at level 2:

- The time to make the nameplates.
- How many could be sold.
- The cost of making the nameplates, including Matt's time (often not considered by learners).
- The price to charge.

Key Stage 4 references (J562)

FB11 Bivariate data	
11.1 - Use charts and correlation	Candidates should be able to: a) Draw line graphs for time series; b) Interpret time series.

FA13 General data handling	
13.1 - Understand and use statistical problem solving process/handling data cycle	Candidates should be able to: a) carry out each of the **four** aspects of the handling data cycle to solve problems: i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed; ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources; iii) process and represent the data: turn the raw data into usable information that gives insight into the problem; b) Interpret and discuss the data: answer the initial question by drawing conclusions from the data.
13.2 - Experimenting	Candidates should be able to: a) discuss how data relate to a problem, identify possible sources of bias and plan to minimise it; b) identify key questions that can be addressed by statistical methods; c) design an experiment or survey and decide what primary and secondary data to use; d) design and use data-collection sheets for grouped discrete and continuous data; e) gather data from secondary sources, including printed tables and lists from ICT-based sources;
13.3 - Processing	Candidates should be able to: a) draw and produce pie charts for categorical data, and diagrams for continuous data, frequency diagrams (bar charts, frequency polygons and fixed interval histograms) and stem and leaf diagrams; b) calculate mean, range and median of small data sets with discrete then continuous data;
13.4 - Interpreting	Candidates should be able to: a) look at data to find patterns and exceptions; b) interpret a wide range of graphs and diagrams and draw conclusions; c) interpret social statistics including index numbers; and survey data; d) compare distributions and make inferences, using the shapes of distributions and measures of average and range; e) understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better population characteristics.

FA4 Ratio	
4.1 - Use ratio notation, including reduction to its simplest form and its various links to fraction notation	Candidates should be able to: a) use ratio notation, including reduction to its simplest form; b) know its various links to fraction notation.

Resources Needed

Graph paper, calculators, rulers and pencils

Misconceptions and common errors

Weaker learners often fail to plan scales.

Common errors are:

- to fail to have equal horizontal spaces between plots
- to create vertical scales that are difficult to use
- to make statements but fail to support them with evidence
- to only consider some of the evidence.

Statements such as these may be made in the Main Activity:

- "Olivia is almost twice as popular as Ellie" but this may not be supported by any evidence drawn from the data
- "Olivia has been the top or second name since 2006" but other facts such as "only reaching the top five in 2005" or "Ellie has been popular before 2006 but is now declining" may be ignored.

Learners often write at length but fail to communicate ideas clearly. Simple notes and headings are often preferable to prose.

"I worked out the average positions. These are
Chloe 3.7
Emily 2.4 ……… and used them to find the number of nameplates by multiplying each by 5 and rounding up.
Chloe 3.7 x 5 = (18.5) =19"
Is preferable to a lengthy essay.

Previously set Functional Mathematics questions

None previously set. The closest comparison is June 2009, Level 1

Asha finds this information on **rainfall** (in millimetres) in the two cities.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rome	80	71	69	67	52	34	16	24	69	113	111	97
New Delhi	23	20	15	10	15	68	200	200	123	19	3	10

Asha says, “I don’t want to go to New Delhi because it’s very wet”.

(c) Draw a chart on page 11 that Asha could use to compare rainfall in New Delhi and Rome.

Use your chart to decide if Asha’s reason for not going to New Delhi is correct.

The Principal Examiner commented:

Part (c) Many good answers were seen. Candidates used the whole of the graph paper, drew good dual bar charts or line graphs and provided a key.
Interpreting the data proved more problematic with one mark from two being the mode. Many identified that New Delhi was wetter than Rome in June, July and August but failed to note that Rome had more rain throughout the year.
Weaker candidates misinterpreted the data as temperature, drew graphs without the aid of a ruler and demonstrated lamentable graphical accuracy. In some cases they redrew a table of values.
Common errors were to omit a title for the vertical scale or to omit a key.

Starter Activity: Tallying and Graphs (Time Series)

This short task is designed to allow discussion to take place, based on some real data.

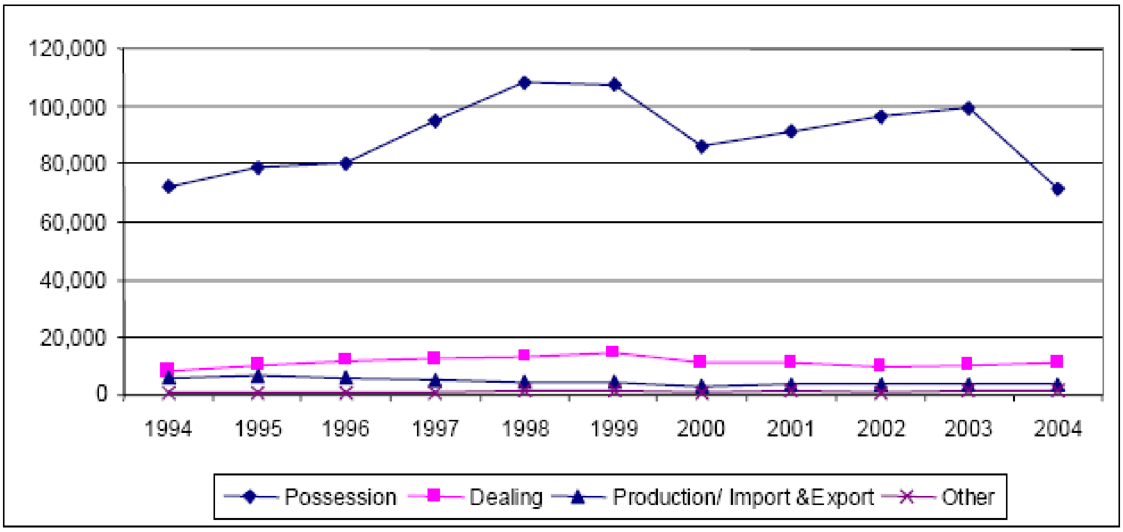
The teacher would project the graph and allow pairs of learners to consider their responses.

Management of the responses would involve separating "hearsay" from evidence in the graph and supporting statements with appropriate evidence.

Consequently, "I agree" is unhelpful but noting that the trend is downwards but is still only back to the level of 1994 after a significant rise from 1996 to 2003, is of more use.

3. In 2004 the total number of drug offences dealt with in England and Wales by the police, courts and HM Revenue and Excise fell to 105,570, following the rise in 2003 to 133,970.

Figure 1: Number of known drug offenders by offence type, England and Wales, 1994-2004



Source: Home Office

I see that the number of drug offences is going down. That's good!

Do you agree with this statement?

Main Activity: Tallying and Graphs (Time Series) - Level 1

Matt has a small woodwork business.

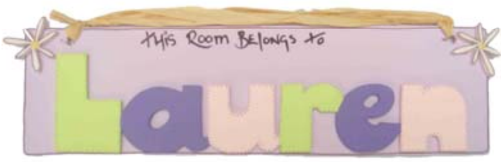
He thinks that he could use some of the small pieces of wood that are left over to make nameplates for doors.

www.hickorydickory.co.uk/

He plans to make some plates for people to see when they come to his workshop.

He wants to sell them from his workshop so he wants to be sure that he chooses popular names and makes the right amount of each one.

He makes these girl's nameplates to start with.



Draw a graph to show how the popularity of girl's names has changed since 1998 (See resource sheet A).

- What is the best sort of graph to use?
- What names will you use?
- Plan your scales carefully.
- What does your graph tell you about the popularity of these girl's names?

1 What name has the highest, consistent, popularity rating?
2 Has Matt chosen the right nameplates to make?
3 What names and what quantities of each should he make?

Main Activity: Tallying and Graphs (Time Series) - Level 2

Matt has a small woodwork business.

He thinks that he could use some of the small pieces of wood that are left over to make nameplates for doors.

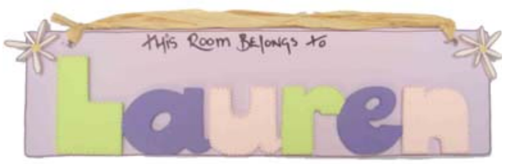
www.hickorydickory.co.uk/

He plans to make some plates for people to see when they come to his workshop.

He wants to sell them from his workshop so he wants to be sure that he chooses popular names and makes the right amount of each one.

He also wants to sell nameplates on the Internet.

He makes these girl's nameplates to start with.



He has the wood to make the nameplates, so this will not cost very much. However, he needs paint, glue, some decoration and raffia to make the nameplates.

It will also take him some time to make each one.

1 Has he chosen the right nameplates to make?

2 What names and what quantities should he make

Extension

3 What price should he charge for each nameplate?

Some items that might be useful when making nameplates

Prices are examples of those that might be found

Item	Description	Cost
Primer Paint	3.75 litres, enough to prime 500 nameplates. Seals any faults and stains in wood and gives a good surface for coloured paint.	£14.50
Eggshell Gloss Colour Paint	2.5 litres, enough to paint 200 nameplates and give overall colour	£11.20
Specialist colour paint	59ml pots of acrylic paint for fine detail on wood. This is needed to paint individual letters. One pot covers one nameplate.	£1.50
Decorative brads	Shapes made to stick to painted wood. One pack of 9 shapes	£0.99
Raffia	100g of plain raffia to use to hang nameplates. Enough for 50 nameplates	£1.50
Raffia	50g of one colour raffia to use to hang nameplates. Enough for 25 nameplates	£1.00
Sticky pads	An Alternative to raffia so that nameplates can be stuck to doors. One pack of 4000 pads	£12

Resource sheet A

These data show the popularity of names, in the UK, for girls and boys born between 1998 and 2009.
The figures in the final column show the number of children registered with the name beside it in 2009 only.
The names are in rank order and include different spellings of the same name (Eg. Emily and Emilee)

Rank	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	Number registered
1	CHLOE	CHLOE	CHLOE	CHLOE	CHLOE	Emily	Emily	Jessica	Olivia	Grace	Isabel	OLIVIA	5317
2	EMILY	EMILY	EMILY	EMILY	EMILY	Ellie	Ellie	Emily	Grace	Ruby	Olivia	RUBY	4924
3	MEGAN	MEGAN	MEGAN	MEGAN	JESSICA	Chloe	Jessica	Sophie	Jessica	Olivia	Emily	EMILY	4874
4	JESSICA	OLIVIA	CHARLOTTE	JESSICA	ELLIE	Jessica	Sophie	Olivia	Ruby	Emily	Sophie	GRACE	4773
5	SOPHIE	SOPHIE	JESSICA	SOPHIE	SOPHIE	Sophie	Chloë	Chloë	Emily	Jessica	Lily	JESSICA	4667
6	CHARLOTTE	CHARLOTTE	LAUREN	LAUREN	MEGAN	Megan	Lucy	Ellie	Sophie	Sophie	Chloe	CHLOE	4601
7	HANNAH	LAUREN	SOPHIE	CHARLOTTE	CHARLOTTE	Lucy	Olivia	Grace	Chloe	Chloe	Jessica	SOPHIE	4378
8	LAUREN	JESSICA	OLIVIA	HANNAH	LUCY	Olivia	Charlotte	Lucy	Lucy	Lily	Grace	LILY	4009
9	REBECCA	REBECCA	HANNAH	OLIVIA	HANNAH	Charlotte	Katie	Charlotte	Lily	Ella	Ella	AMELIA	3437
10	LUCY	HANNAH	LUCY	LUCY	OLIVIA	Hannah	Megan	Katie	Ellie	Amelia	Abigail	EVIE	3275
11	AMY	BETHANY	GEORGIA	ELLIE	LAUREN	Katie	Grace	Ella	Ella	Lucy	Mia	MIA	3113
12	GEORGIA	LUCY	REBECCA	AMY	KATIE	Ella	Hannah	Megan	Charlotte	Charlotte	Hannah	ELLA	3023
13	KATIE	GEORGIA	BETHANY	KATIE	AMY	Grace	Amy	Hannah	Katie	Ellie	Lucy	CHARLOTTE	2937
14	BETHANY	AMY	AMY	GEORGIA	MOLLY	Mia	Ella	Amelia	Mia	Mia	Ruby	LUCY	2871
15	EMMA	KATIE	ELLIE	REBECCA	HOLLY	Amy	Mia	Ruby	Hannah	Evie	Isabella	MEGAN	2515
16	OLIVIA	ELLIE	KATIE	MOLLY	ELLA	Holly	Lily	Lily	Amelia	Hannah	Emma	ELLIE	2482
17	COURTNEY	EMMA	EMMA	BETHANY	BETHANY	Lauren	Abigail	Amy	Megan	Megan	Amelia	ISABELLE	2459
18	SHANNON	COURTNEY	ABIGAIL	EMMA	REBECCA	Emma	Emma	Mia	Amy	Katie	Sophia	ISABELLA	2421
19	ELEANOR	ELEANOR	MOLLY	HOLLY	GRACE	Molly	Amelia	Abigail	Isabella	Isabella	Ava	HANNAH	2331
20	JADE	ABIGAIL	GRACE	ELLA	MIA	Abigail	Molly	Millie	Millie	Isabelle	Evie	KATIE	2318

Rank	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	
1	JACK	JACK	JACK	JACK	JACK	Jack	Jack	Jack	Jack	Jack	Jack	JACK	8007
2	THOMAS	THOMAS	THOMAS	THOMAS	JOSHUA	Joshua	Joshua	Joshua	Thomas	Thomas	Oliver	OLIVER	7413
3	JAMES	JAMES	JAMES	JOSHUA	THOMAS	Thomas	Thomas	Thomas	Joshua	Oliver	Joshua	THOMAS	6054
4	DANIEL	JOSHUA	JOSHUA	JAMES	JAMES	James	James	James	Oliver	Joshua	Charlie	HARRY	6006
5	JOSHUA	DANIEL	DANIEL	DANIEL	DANIEL	Daniel	Daniel	Oliver	Harry	Harry	Daniel	JOSHUA	5713
6	MATTHEW	MATTHEW	HARRY	HARRY	BENJAMIN	Oliver	Samuel	Daniel	James	Charlie	Thomas	ALFIE	5557
7	SAMUEL	SAMUEL	SAMUEL	SAMUEL	WILLIAM	Benjamin	Oliver	Samuel	William	Daniel	James	CHARLIE	5285
8	CALLUM	JOSEPH	JOSEPH	JOSEPH	SAMUEL	William	William	William	Samuel	William	Harry	DANIEL	5185
9	JOSEPH	CALLUM	MATTHEW	MATTHEW	JOSEPH	Samuel	Benjamin	Harry	Daniel	James	Ethan	JAMES	5168
10	JORDAN	WILLIAM	CALLUM	LEWIS	OLIVER	Joseph	Joseph	Joseph	Charlie	Alfie	William	WILLIAM	5167
11	CONNOR	RYAN	LUKE	LUKE	HARRY	Harry	Harry	Benjamin	Benjamin	Samuel	Dylan	SAMUEL	4620
12	RYAN	LUKE	WILLIAM	OLIVER	MATTHEW	Matthew	Matthew	Charlie	Joseph	George	Alfie	GEORGE	4209
13	LUKE	LEWIS	LEWIS	WILLIAM	LUKE	Lewis	Lewis	Luke	Callum	Joseph	Samuel	JOSEPH	3760
14	WILLIAM	HARRY	OLIVER	BENJAMIN	LEWIS	Luke	Ethan	Matthew	George	Benjamin	George	LEWIS	3482
15	HARRY	JORDAN	RYAN	CALLUM	GEORGE	Ethan	Luke	Callum	Jake	Ethan	Max	ETHAN	3445
16	BENJAMIN	BENJAMIN	BENJAMIN	GEORGE	CALLUM	George	Charlie	Jake	Alfie	Lewis	Matthew	MOHAMMED	3423
17	GEORGE	LIAM	GEORGE	ADAM	ADAM	Adam	George	Ethan	Luke	Mohammed	Jacob	DYLAN	3370
18	LEWIS	GEORGE	LIAM	RYAN	ETHAN	Alfie	Callum	George	Matthew	Jake	Jayden	BENJAMIN	3274
19	ALEXANDER	ALEXANDER	JORDAN	JAKE	ALEXANDER	Callum	Alexander	Lewis	Ethan	Dylan	Lucas	ALEXANDER	3213
20	OLIVER	ADAM	ADAM	ADAM	RYAN	Alexander	Mohammed	Alexander	Lewis	Jacob	Alexander	JACOB	3128

Chloe	1	1	1	1	1	3	5	5	7	7	6	6	3.666666667
Emily	2	2	2	2	2	1	1	2	5	4	3	3	2.416666667
Megan	3	3	3	3	6	6	10	12	17	17		15	8.636363636
Jessica	4	8	5	4	3	4	3	1	3	5	7	5	4.333333333
Sophie	5	5	7	5	5	5	4	3	6	6	4	7	5.166666667
Jack	1	1	1	1	1	1	1	1	1	1	1	1	1
Thomas	2	2	2	2	3	3	3	3	2	2	6	3	2.75
James	3	3	3	4	4	4	4	4	6	9	7	9	5
Daniel	4	5	5	5	5	5	5	6	9	7	5	8	5.75
Joshua	5	4	4	3	2	2	2	2	3	4	3	5	3.25

Lesson Plan: Tallying and Graphs (Time Series)

Time	**Content**
5 - 10 minutes	Starter activity – Give out the task sheet and discuss Matt's choice of names. Would anyone in the class be able to buy one of these names? What names would people in the class make and why? (Create list) Extension: What other costs could be involved in making and selling the nameplates? (Make a list that will feed into homework.)
10 - 15 minutes	Give out Resource sheet (Rank order names) Do these names agree with our list? (Boys and girls) What trends do you see in the popularity of names? (Learners write some down, then discuss) How could we show this in another way? (You could link to www.basingstoke.gov.uk/.../Ethnicity/2006.htm and view the time series regarding changes in ethnic populations and spend a few minutes interpreting the graphs. Many other examples are available)
25–30 minutes	Choose some names from the top of the list and track their popularity (by using their ranks) to see any trends. Describe the trends. How can Matt use these results to decide which nameplates to make? (You could also find average ranks for the top …names) Could you use the final column to suggest how many nameplates to make? Write your recommendations and refer to your results from the graphs (and calculations).
5 minutes	Draw the class together and, briefly, compare recommendations. Emphasise need for supporting evidence Homework: Research costs of other items for use in making the nameplates. (Check that learners have lists of give out blank Resource Sheet to complete.)

This extension is optional. Time could be spent refining the work of analysing, and interpreting the data.

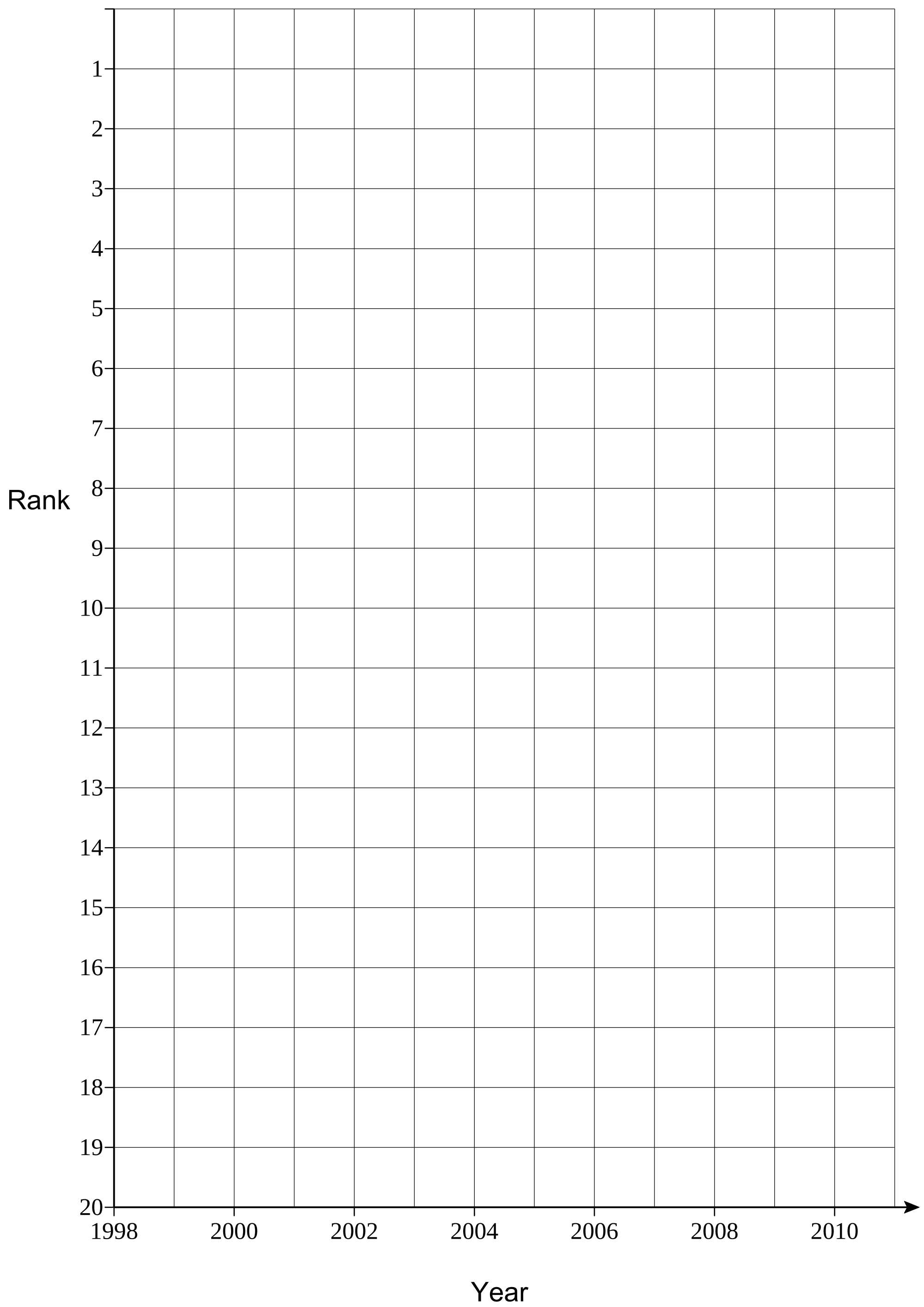
Time	**Content**
5 - 10 minutes	Display anonymous examples of comments and consider how well data has been used to make recommendations
10 - 15 minutes	Starter activity – Establish what costs learners have found. Give out Resource Sheet Costs, if needed Consider amounts required How much should Matt charge for his time?
25–30 minutes	Learners work (in pairs) to establish costs of making each nameplate and recommend a price. Estimate the number that might be sold. Write up results
5 minutes	Feedback from some learner (pairs) about the price likely to be charged, how many he could sell and profit. How much income could he make? What other costs could Matt have? (Heating, light, rent, other worker to be paid etc)

Resource sheet

Research the costs of these items.

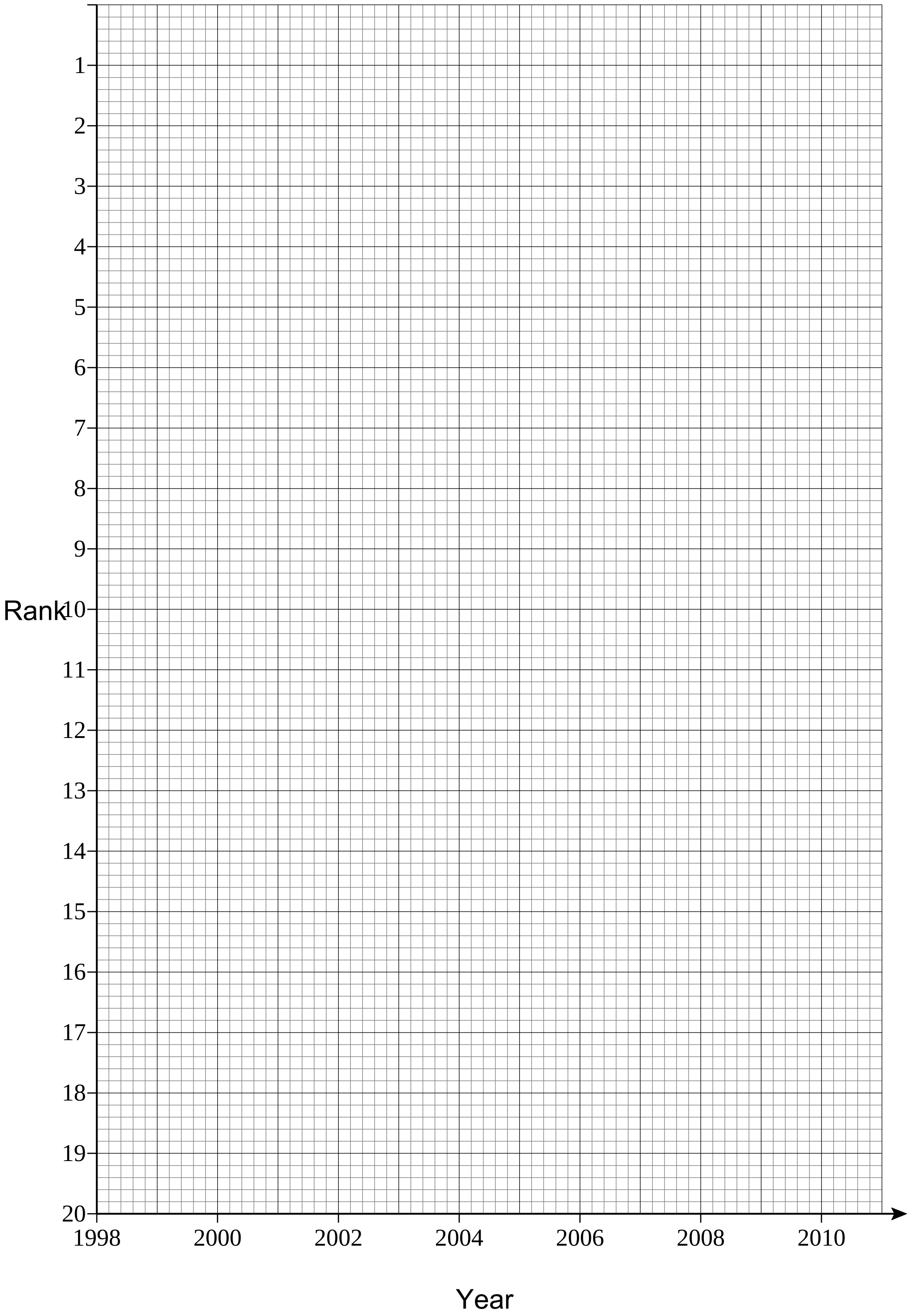
Item	Description	Cost
Primer Paint		
Eggshell Gloss Colour Paint		
Specialist colour paint		
Decorative brads		
Raffia		
Sticky pads		

Time Series Graph showing popularity of names in UK, 1998 to 2009



Resource sheet

Time Series Graph showing popularity of names in UK, 1998 to 2009



Some possible outcomes

These names are not good choices. Only Lauren has been in the top 20 names and then not since 2003.

Ranks and mean ranks for top FIVE only.

Name	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	Mean
Chloe	1	1	1	1	1	3	5	5	7	7	6	6	3.67
Emily	2	2	2	2	2	1	1	2	5	4	3	3	2.42
Megan	3	3	3	3	6	6	10	12	17	17		15	8.64
Jessica	4	8	5	4	3	4	3	1	3	5	7	5	4.33
Sophie	5	5	7	5	5	5	4	3	6	6	4	7	5.17
Jack	1	1	1	1	1	1	1	1	1	1	1	1	1
Thomas	2	2	2	2	3	3	3	3	2	2	6	3	2.75
James	3	3	3	4	4	4	4	4	6	9	7	9	5
Daniel	4	5	5	5	5	5	5	6	9	7	5	8	5.75
Joshua	5	4	4	3	2	2	2	2	3	4	3	5	3.25

Comments such as

Chloe has a mean rank of 3.67 but the recent popularity is declining.
Looking at numbers of registrations difference between top and bottom (of top 5) only 5317 – 4667 = 650

In order of means Emily, Chloe, Jessica, Sophie, Megan

There maybe local effects.

He should make only names in the top 10 as samples.
Ratios could be …

Rank	Name	Number registered	Number of nameplates made (1 for every 1000)	Number of nameplates made (1 for every 500)
1	CHLOE	5317	5	11
2	EMILY	4924	5	10
3	MEGAN	4874	5	10
4	JESSICA	4773	5	10
5	SOPHIE	4667	5	9
6	CHARLOTTE	4601	5	9
7	HANNAH	4378	4	9
8	LAUREN	4009	4	8
9	REBECCA	3437	3	8
10	LUCY	3275	3	7

Tallying and Graphs (Block Graphs) (Level 1 and 2)

Introduction

The aim of the activity is to give learners practise in drawing graphs to represent real data and draw conclusions. The purpose of the graph is to give a visual comparison of the relative popularity of the different names given to boys in 2009. This should be used to decide on the numbers of each nameplate that could be made.

The task could be introduced to the whole group and issues discussed.

- What sort of graph is most appropriate to display the information?
- Should all the names be included on the graph?
- Patterns in the data.
- What scale should be used for the frequency axis of the graph?
- Level 2, what accuracy could be plotted?
- The purpose of the graph.
- Appropriate titles.

After drawing the graph learners might work in pairs to decide
(i) Which names to recommend and
(ii) How many of each nameplate should be made.
Learners should then write **clear** conclusions, not in flowing prose but as brief notes with justifications for proportions from the data. These should be presented to the group and decisions challenged.

Level 2 learners may note that these results are for 2009 and may reflect the needs of newborn babies but do not inform us of patterns for people born before 2009. Further research is needed to find patterns for earlier years. This could be a good ICT activity.

Learners may want to consider other issues such as:

- Is the pattern of name popularity reflected in their school (college)?
- Design a survey to gather data and compare with these figures
- What about girl's names?
- The time to make the nameplates
- How many could be sold
- The cost of making the nameplates, including Joseph's time (often not considered by learners)
- The price to charge.

FA13 General data handling	
13.1 - Understand and use statistical problem solving process/handling data cycle	Candidates should be able to: a) carry out each of the **four** aspects of the handling data cycle to solve problems: i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed; ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources; iii) process and represent the data: turn the raw data into usable information that gives insight into the problem; b) Interpret and discuss the data: answer the initial question by drawing conclusions from the data.
13.2 - Experimenting	Candidates should be able to: a) discuss how data relate to a problem, identify possible sources of bias and plan to minimise it; b) identify key questions that can be addressed by statistical methods; c) design an experiment or survey and decide what primary and secondary data to use; d) design and use data-collection sheets for grouped discrete and continuous data; e) gather data from secondary sources, including printed tables and lists from ICT-based sources;
13.3 - Processing	Candidates should be able to: a) draw and produce pie charts for categorical data, and diagrams for continuous data, frequency diagrams (bar charts, frequency polygons and fixed interval histograms) and stem and leaf diagrams; b) calculate mean, range and median of small data sets with discrete then continuous data;
13.4 - Interpreting	Candidates should be able to: a) look at data to find patterns and exceptions; b) interpret a wide range of graphs and diagrams and draw conclusions; c) interpret social statistics including index numbers; and survey data; d) compare distributions and make inferences, using the shapes of distributions and measures of average and range; e) understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better population characteristics.

FA4 Ratio	
4.1 - Use ratio notation, including reduction to its simplest form and its various links to fraction notation	Candidates should be able to: a) use ratio notation, including reduction to its simplest form; b) know its various links to fraction notation.

Resources Needed

Graph paper, calculators, rulers and pencils

Misconceptions and common errors

Weaker learners often fail to plan scales.
Common errors are:

- To fail to have equal width columns or equal horizontal spaces between plots.
- To create difficult to use continuous vertical scales.
- To plot frequencies at equally spaced points on the vertical axis, regardless of the magnitude of each frequency. Thus 200, 500, 520, 810 may each be 2cm apart and simply written against the height of the column on the vertical scale.

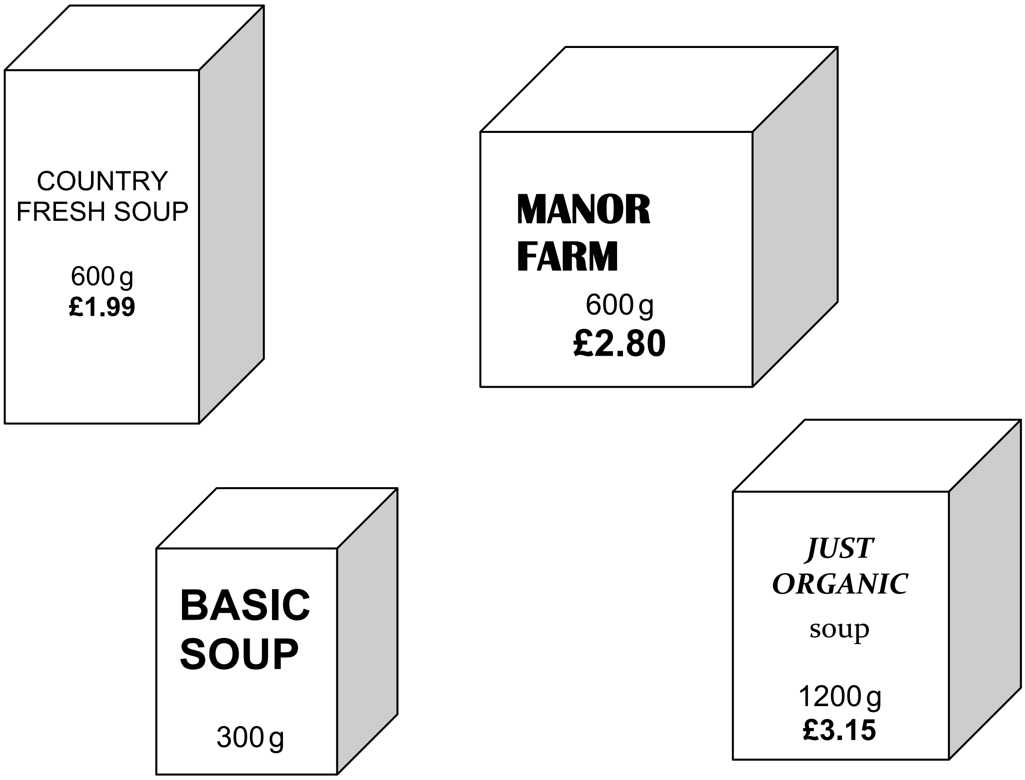
Learners also omit to refer clearly to evidence. Thus Jack is almost twice as popular as George but this may not be reflected in the relative number of nameplates or referred to in the justification. Learners often write at length but fail to communicate ideas clearly. Simple notes and headings are often preferable to prose.

"I decided to recommend one plate for every 500 names and rounded up the results."
Jack 8007 ÷ 500 = 16
Oliver 7423 ÷ 500 = 15 etc
Is preferable to a lengthy essay.

Previously set Functional Mathematics questions
May 2009 Level 1

Jamie's ingredients cost him £4.36.

While he is shopping, Jamie looks at ready-made leek and potato soups and makes a note of the different prices. All the cartons say that 300 g of soup is enough for 1 person.



Jamie thinks that his leek and potato soup is much cheaper than the ready-made soups. He decides to draw a bar chart to show his friends.

(b) Draw a bar chart on the grid opposite showing the cost of each soup for 12 people and use the evidence to decide whether Jamie is right.

The Principal Examiner commented:

Candidates sometimes, but not always, showed that they needed to multiply soup packet prices by an appropriate number to standardise the cost on 12 people.
Some candidates were explicit about the number of people each packet would feed. Far too many, however, either wrote down the prices they thought were correct for 12 people or drew the graph without any evidence of the costs represented. Others drew a graph showing the prices given on the packets.

Common errors were;

- To multiply all prices by 12 (assuming one packet per person).
- To assume **one** packet of each soup was needed.
- To fail to respond to the request to consider Jamie's assertion.
- To fail to **label** the axes.
- To use their scale incorrectly.
- To omit Jamie's soup from the graph.

Starter Activity 1: Tally - Notes

The starter could be used to refresh ideas of recording using tallies.
Students could work in pairs.

Errors made.

- Not grouping in fives (Adam).
- Grouping in tens (William).
- Wrong total (14 for Luke and should be 13)
- Wrong total overall (52 on given tallies).

In response to Barry's assertion;

- We have no idea whether the tally marks accurately reflect the student responses.
- The totals could be correct and the tallies incorrect (less likely)
- Some students might have given her two "favourites" and she could have counted these.
- Some students may not have preferred any of the names.

It would be sensible to discuss the fact that there could be many "correct" answers on the evidence given.

A blank copy of the table for student use

Name	Tally	Frequency
Ethan		
Luke		
William		
Jordan		
Adam		
	Total	

Starter Activity 1: Tally

Jenny has made this record of the popularity of some names with 50 students at her college.

She has made some errors.

What errors has she made?

Name	Tally	Frequency
Ethan	~~||||~~ | |	7
Luke	| | | | | | | | | | | | |	14
William	~~|||||||||~~ | | | |	14
Jordan	| | | |	4
Adam	~~||||~~ | | | | | | | |	13
	Total	50

I don't think I can say ALL the errors she made.

Is Barry right?

Starter Activity 2: Tally - Notes

A copy of the data for each learner is essential.

The purpose is to practise tallying **accurately** for a reasonably large data set and to draw conclusions.
Learners need to work carefully to complete a table.
Use "Five bar gate" method.
They should decide their group sizes and could refine these but, if the purpose is to check accuracy, then the use of the given table is a good idea.
Checking should be discussed and picking up obvious errors. (Total ≠ 120 etc)

Question 1

- The actual area may be in the region of 120 to 160 cm^2 (The mean is 155 cm^2 to 3sf)
- There is no way to know the precise area from the data.

Question 2

- This is open for discussion and could be based on the area being given as 155 cm^2. This could give rise to a scoring system based on ranges from the (given) mean.
- Discussion of the shape of the distribution (single hump, centered and small standard deviation) but no way of knowing how well they estimated.

It would be sensible to discuss the fact that there could be many "correct" answers on the evidence given.

A survey could be conducted in the centre to determine the accuracy of estimates made. (They are likely to be MUCH poorer than these and show very marked deviations.)

A table for student use (if required)

Estimate	Tally	Frequency
0 - 59		
60 - 79		
80 - 99		
100 - 119		
120 - 139		
140 - 159		
160 - 179		
180 - 199		
200 - 219		
220 +		
	Total	

Starter Activity 2: Tally – Resource sheet

Estimate	Tally	Frequency
	Total	

Starter Activity 2: Tally – Data set

A copy of the data set

90	141	180	151	112	130
100	135	168	201	146	132
189	152	126	141	165	140
170	188	136	145	139	126
41	100	129	156	146	146
141	111	134	160	183	157
158	162	144	120	125	125
130	132	147	150	159	154
122	129	139	146	132	133
147	165	146	158	165	119
143	121	1015	166	183	620
156	149	148	146	175	157
118	170	138	129	148	166
169	137	142	134	93	167
21	147	160	141	177	100
139	177	184	140	146	131
136	136	145	175	250	122
166	138	136	160	151	133
57	145	190	162	140	125
148	150	164	137	121	169

Starter Activity 3: Tallying and graphs (Block Graphs)

Brian asked 120 people to estimate the area of a rectangle.

These are the estimates people give to Brian for the area of the rectangle.

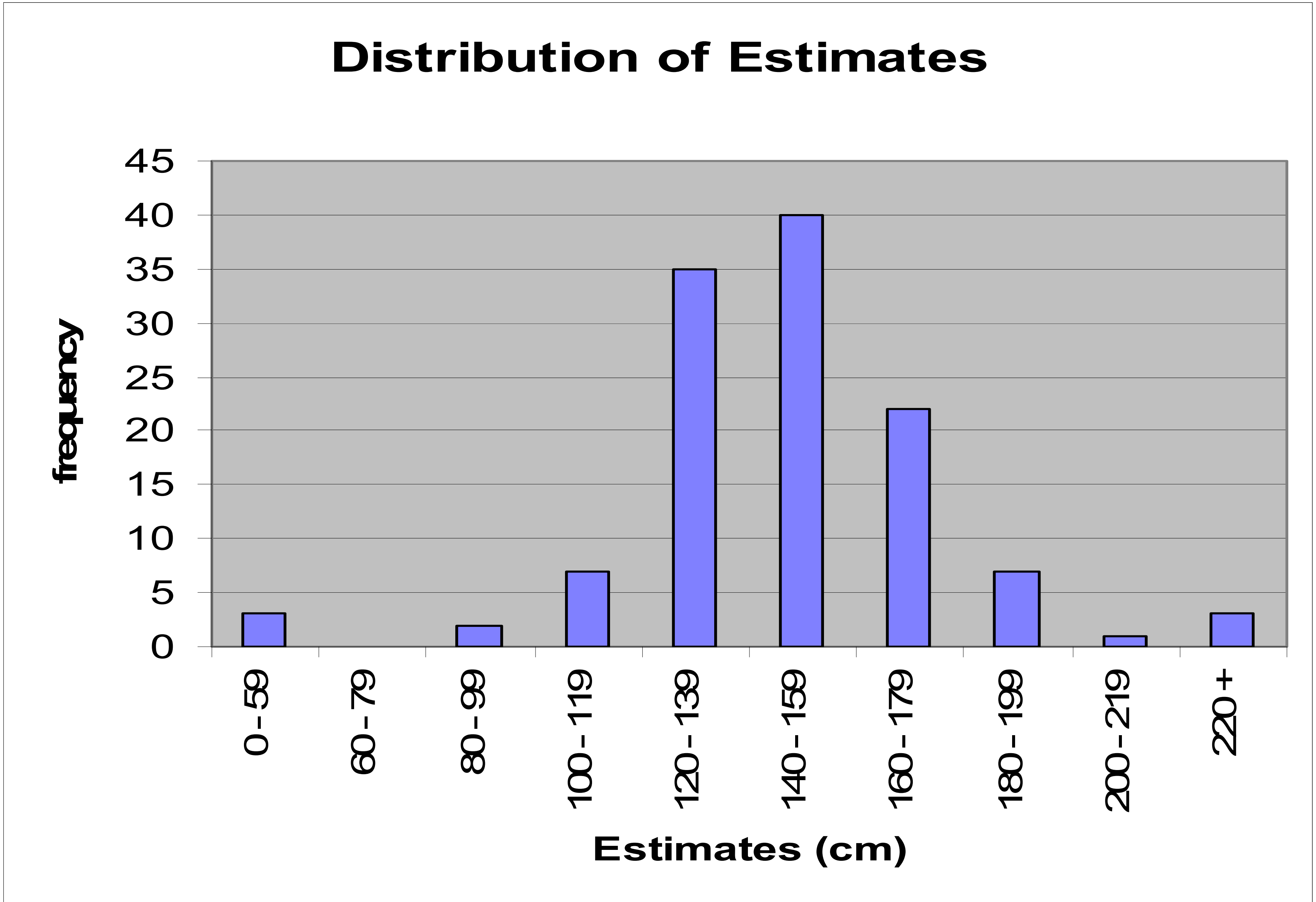
All estimates are in cm^2

90	141	180	151	112	130
100	135	168	201	146	132
189	152	126	141	165	140
170	188	136	145	139	126
41	100	129	156	146	146
141	111	134	160	183	157
158	162	144	120	125	125
130	132	147	150	159	154
122	129	139	146	132	133
147	165	146	158	165	119
143	121	1015	166	183	620
156	149	148	146	175	157
118	170	138	129	148	166
169	137	142	134	93	167
21	147	160	141	177	100
139	177	184	140	146	131
136	136	145	175	250	122
166	138	136	160	151	133
57	145	190	162	140	125
148	150	164	137	121	169

1 What do you think was the area of the rectangle these people were shown?

2 Use the data and decide how good at estimating these people are.

Block graph showing the distribution of estimates for the rectangle in Starter 2, Tally



Main Activity: Tallying and Graphs (Block Graphs)

Joseph is a carpenter.

He decides to make wooden nameplates to sell in his shop.

Joseph has to decide:

- Which names to make nameplates for and
- How many of each nameplate to make.

This table shows the most popular names for newborn boys in 2009.

2009	Number of registered names	**2009**	Number of registered names
JACK	8007	SAMUEL	4620
OLIVER	7413	GEORGE	4209
THOMAS	6054	JOSEPH	3760
HARRY	6006	LEWIS	3482
JOSHUA	5713	ETHAN	3445
ALFIE	5557	MOHAMMED	3423
CHARLIE	5285	DYLAN	3370
DANIEL	5185	BENJAMIN	3274
JAMES	5168	ALEXANDER	3213
WILLIAM	5167	JACOB	3128

- Explain any decisions you make.
- Recommend which names Joseph should make nameplates for and how many of each nameplate he should make.
- Justify your decisions by referring to the information.
- Explain why using this information might mean that Joseph does not make the right nameplates.
- Draw a graph that shows the popularity of different boy's names.

Averages (London Marathon) - Level 1

Introduction

The aim of the activity is to give learners practise in extracting information from tables, calculating means and range and use these to answer a simple question.

The task is adapted from Level 2 and is difficult for Level 1 as it involves continuous data and time. (Continuous data is a Level 2 requirement. However a simplified version with only hours and minutes may resolve some issues and strategies may be considered to assist Level 1 learners, as given in the starter. As the hour component is common to all runners, at this level, the only figure that needs averaging is for minutes.)

18 to 39 years

Name	Half time (h : m)	Finish time (h : m)
Kebede, Tsegaye (ETH)	1:03	2:05
Mutai, Emmanuel (KEN)	1:03	2:06
Gharib, Jaouad (MAR)	1:03	2:07
Bouramdane, Abderrahime (MAR)	1:03	2:08
Kirui, Abel (KEN)	1:03	2:08
Gomes dos Santos, Marilson (BRA)	1:03	2:09
Tadese, Zersenay (ERI)	1:03	2:12
Lemoncello, Andrew (GBR)	1:05	2:14
Kifle, Yonas (ERI)	1:03	2:15
Jones, Andi (GBR)	1:06	2:17

40 to 44 years

Bilton, Darran E (GBR)	1:11	2:25
Cope, Nicholas (GBR)	1:11	2:26
Vautier, Stephane (FRA)	1:12	2:28
Weir, Andrew P (GBR)	1:14	2:30
Lincoln, Wayne (GBR)	1:14	2:30
Richmond, Kenny (GBR)	1:14	2:32
Smalls, Allen (GBR)	1:17	2:33
Berg, Jens – Kristian (NOR)	1:15:	2:35
Knight, Geoffrey P (GBR)	1:15	2:35
Freeman, David J (GBR)	1:17	2:36

45 to 49 years

Crampton, Ian (GBR)	1:13	2:30
Larripa, Eric (FRA)	1:15	2:33
Rackham, Nigel D (GBR)	1:17	2:36
Downs, Rob H (GBR)	1:16	2:38
Hatton, Michael (GBR)	1:19	2:39
Foster, Philip A (GBR)	1:17	2:40
Waby, Paul (GBR)	1:19	2:40
Gee, Sarah R (GBR)	1:19	2:40
Reilly, Leonard J (GBR)	1:21	2:40
Croasdale, Mark J (GBR)	1:15	2:40

The task could be introduced to the whole group and issues discussed. Questions have been posed that lead learners to work out means and ranges and compare these. However, many learners find the process of calculating relatively easy (and mechanical) but cannot interpret. They will need to express their ideas to the group (or teacher) and be corrected on poor use of the measures of location.
Ideas could be discussed about the limited amount of data available and how this could be remedied.

This task is ideal for ICT and the spreadsheets may be used and added to.

The answers to the questions are:

18 to 39 years	Mean (half way)	1h 3m 37s	Range	2m 39s
	Mean (complete)	2h 10m 0s	Range	11m 19s
40 to 44 years	Mean (half way)	1h 14m 3s	Range	6m 34s
	Mean (complete)	2h 30m 59s	Range	10m 48s
45 to 49 years	Mean (half way)	1h 17m 3s	Range	7m 45s
	Mean (complete)	2h 37m 35s	Range	10m 19s

Learners could work in pairs if ICT is not used or individually to process data on spreadsheets. Further data may be downloaded from this website.

http://results-2010.virginlondonmarathon.com/2010/

The data will need to be downloaded sheet by sheet and pasted into the spreadsheet.

It is important that outcomes are discussed and learners justify their assertions with use of data.

Functional Standards

R1	Understand that means give representative values for groups of runners and also refer to tables to compare individual runners
R2	Extract **relevant** information from the tables
R3	Know correct methods to calculate mean and range and decide how to use these results and individual results to answer the question, "Are older runners slower than younger ones?"
A1	Use correct methods to calculate mean and range as well as compare relevant individual runner's results.
A2	Respond positively when asked to repeat calculations or check that results seem sensible, or not.
I1	Answer the question "Are older runners slower than younger ones?" with more than a "Yes/No" answer. Use their results and explain how they inform their answer. Make some comment on such things as accuracy, limited data etc.

FA13 General data handling	
13.1 - Understand and use statistical problem solving process/handling data cycle	Candidates should be able to: a) carry out each of the **four** aspects of the handling data cycle to solve problems: i) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources; iii) process and represent the data: turn the raw data into usable information that gives insight into the problem; b) Interpret and discuss the data: answer the initial question by drawing conclusions from the data.
13.2 - Experimenting	Candidates should be able to: a) discuss how data relate to a problem, identify possible sources of bias and plan to minimise it; b) identify key questions that can be addressed by statistical methods; c) design an experiment or survey and decide what primary and secondary data to use; d) design and use data-collection sheets for grouped discrete and continuous data; e) gather data from secondary sources, including printed tables and lists from ICT-based sources;
13.3 - Processing	Candidates should be able to: a) draw and produce pie charts for categorical data, and diagrams for continuous data, frequency diagrams (bar charts, frequency polygons and fixed interval histograms) and stem and leaf diagrams; b) calculate mean, range and median of small data sets with discrete then continuous data;
13.4 - Interpreting	Candidates should be able to: a) look at data to find patterns and exceptions; b) interpret a wide range of graphs and diagrams and draw conclusions; c) interpret social statistics including index numbers; and survey data; d) compare distributions and make inferences, using the shapes of distributions and measures of average and range; e) understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better population characteristics.

Resources Needed

Calculators, computer terminals

Misconceptions and common errors

- Candidates frequently fail to take note of headings in tables.
- They transcribe data from the tables incorrectly.
- Candidates often do not show the working they have used to obtain answers and so it is often difficult to know whether they have used correct figures or not.
- Answers are often poorly expressed. Learners often write at length but fail to communicate ideas clearly. Simple notes and headings are often preferable to prose
- Candidates can apply mechanical processes but often do not understand the outcome. Means may be calculated but their significance as a representative value is often poorly understood.

Previously set Functional Mathematics questions Level 2

Abasi and Owen are talking about the countries that won medals in the 2006 Winter Paralympics in Turin.

(A table of data was supplied)

They collect information about some countries that won medals.

The information that Abasi and Owen collected is shown in the Resource Booklet.

Use the information in the table to investigate whether:

(a) wealthy countries won more gold medals,

(b) big countries won more gold medals.

You must show your calculations. State how you have used them and any charts you might draw to get your answer.

The Principal Examiner commented:

Most candidates, who attempted this task, drew graphs of some form or another. Only a very small number used averages and these were to support their graphical approaches. Some understood that they were to explore the connection (or not) between the three fields - country, number of gold medals and the GDP. A small number tried to categorise countries into rich/poor or big/small by looking for natural break-points this often split the countries into 4 and 11.
Unfortunately there were only a small number of scatter graphs drawn, but these tended to get almost full credit. However, those who drew bar or line graphs merely drew the countries in the order they were in the resource booklet and gained minimal credit.

Starter Activity 1: Averages - Level 1 Notes

Marathon

This is a short starter that encourages learners to extract and interpret data from graphs.
A time should be set (say five minutes) and learners work, on their own, to write down their findings.
Discussion should take place afterwards when learners present and justify their answers.
A projection of the graph is essential and learners should be encouraged to stand before the group and present their findings.

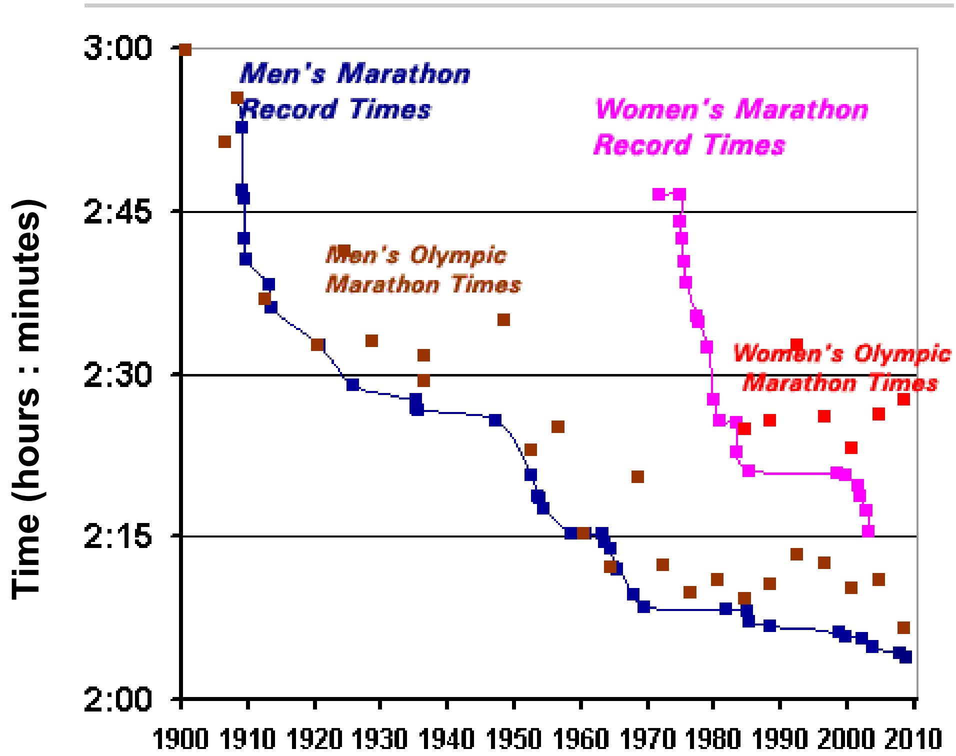
Many statements could be made but probe general statements like “People are quicker” with a, “How do you know?” follow up.

- Scales cannot be read accurately.
- For both genders the trend is downwards.
- Women did not record times (didn’t compete) before the 1970s.
- The rate of improvement for women is more rapid than that for men.
- Few records are set at Olympic games.
- Women, on average, run the marathon more slowly than men taking roughly 12 minutes longer.
- Etc

Starter Activity 1: Averages - Level 1

Marathon

This graph shows the way that the record times for the marathon have changed.



Write down four things that the graph tells you about these marathon record times.

(You can write more if you want!)

You have 4 minutes.

Starter Activity 2: Averages - Level 1 Notes

London Marathon 2010

This is an extensive and challenging starter for Level 1. Depending on the ability of the learners an alternative table could be substituted with times in hours and minutes only.

Place Overall	Name	Age (years)	Half time (h : m)	Finish time (h : m)
1	Kebede, Tsegaye (ETH)	18-39	01:03	02:05
2	Mutai, Emmanuel (KEN)	18-39	01:03	02:06
3	Gharib, Jaouad (MAR)	18-39	01:03	02:06
4	Bouramdane, Abderrahime (MAR)	18-39	01:03	02:07
5	Kirui, Abel (KEN)	18-39	01:03	02:08
6	Gomes dos Santos, Marilson (BRA)	18-39	01:03	02:08
7	Tadese, Zersenay (ERI)	18-39	01:03	02:12
8	Lemoncello, Andrew (GBR)	18-39	01:05	02:13
9	Kifle, Yonas (ERI)	18-39	01:03	02:14
10	Jones, Andi (GBR)	18-39	01:05	02:16

1 *Were these runners in the same order half way through the course?*

2 *How many minutes and seconds more than Kebede did Jones take to run the race?*

3 *How old was Mutai when he came second?*

4 *What was the average time that the first ten runners took to finish the race?*

This does, of course, lose much of the detail that places runners in order and, alternatively, strategies could be discussed to overcome the time issue (Ignore the 2 hours as this is common and work in seconds only or average the minutes and then average the seconds and bring these together at the end.)
Learners often have problems working with time and finding means with time may pose challenges. Using the “angle” key on the calculator (degrees, minutes, and seconds) may help some learners and confuse others.

It is designed to take around 20 minutes to complete but will take longer to discuss.
A copy of the data would be VERY useful to each learner to work from.
The starter is probably best suited to working in pairs and then discussing answers.
It sets up the ideas of using means to represent performances.

Questions 1 to 3 are designed to make learners look at the data and begin to work with time.

Question 1 No not accept a simple Yes/No answer. Always follow up with “How do you know?” and look for justification. The question may be answered, for example, simply by pointing out that Kifle was equal in time with Mutai (and Kirui) at the half way stage but was below them at the finish. Alternatively time could be spent producing a rank order. It is the reasoning and its expression that matters here.

A table of half time positions is shown. (Note: the actual positions at the half way stage cannot be known as runners that finished below 10th may have been ahead of these runners at this stage. This could be a useful question to pose.)

Final position				
9	Kifle, Yonas (ERI)	18-39	01:03:06	02:14:39
5	Kirui, Abel (KEN)	18-39	01:03:06	02:08:04
2	Mutai, Emmanuel (KEN)	18-39	01:03:06	02:06:23
4	Bouramdane, Abderrahime (MAR)	18-39	01:03:07	02:07:33
3	Gharib, Jaouad (MAR)	18-39	01:03:07	02:06:55
6	Gomes dos Santos, Marilson (BRA)	18-39	01:03:07	02:08:46
1	Kebede, Tsegaye (ETH)	18-39	01:03:07	02:05:19
7	Tadese, Zersenay (ERI)	18-39	01:03:07	02:12:03
8	Lemoncello, Andrew (GBR)	18-39	01:05:27	02:13:40
10	Jones, Andi (GBR)	18-39	01:05:45	02:16:38

Question 2 The answer is 11 minutes and 19 seconds

Question 3 cannot, of course, be answered from the given data.

Question 4 is designed to introduce the idea (and challenge) of mean times.
The answer is 2 hours 10 minutes (and 0 seconds)

The starter addresses

Coverage and Range:
- Convert units of measure in the same system
- Extract and interpret information from tables
- Find mean (and range).

Skill standards
- All (A2 will be addressed if learners are encouraged to check whether their answers are sensible.)

Starter Activity 2: Averages - Level 1

London Marathon 2010

This information is about the top ten finishers in the 2010 London Marathon.

Place Overall	Name	Age (years)	Half time (h : m : s)	Finish time (h : m : s)
1	Kebede, Tsegaye (ETH)	18-39	01:03:07	02:05:19
2	Mutai, Emmanuel (KEN)	18-39	01:03:06	02:06:23
3	Gharib, Jaouad (MAR)	18-39	01:03:07	02:06:55
4	Bouramdane, Abderrahime (MAR)	18-39	01:03:07	02:07:33
5	Kirui, Abel (KEN)	18-39	01:03:06	02:08:04
6	Gomes dos Santos, Marilson (BRA)	18-39	01:03:07	02:08:46
7	Tadese, Zersenay (ERI)	18-39	01:03:07	02:12:03
8	Lemoncello, Andrew (GBR)	18-39	01:05:27	02:13:40
9	Kifle, Yonas (ERI)	18-39	01:03:06	02:14:39
10	Jones, Andi (GBR)	18-39	01:05:45	02:16:38

Data from http://results-2010.virginlondonmarathon.com/2010/

1 Were these runners in the same order half way through the course?

2 How many minutes and seconds more than Kebede did Jones take to run the race?

3 How old was Mutai when he came second?

4 What was the average time that the first ten runners took to finish the race?

Main Activity: Averages (London Marathon) - Level 1

London Marathon 2010

Here is the data for the first 10 finishers in the London Marathon 2010 for two different age groups.

18 to 39 years

Name	Half time (h : m : s)	Finish time (h : m : s)
Kebede, Tsegaye (ETH)	1:03:07	2:05:19
Mutai, Emmanuel (KEN)	1:03:06	2:06:23
Gharib, Jaouad (MAR)	1:03:07	2:06:55
Bouramdane, Abderrahime (MAR)	1:03:07	2:07:33
Kirui, Abel (KEN)	1:03:06	2:08:04
Gomes dos Santos, Marilson (BRA)	1:03:07	2:08:46
Tadese, Zersenay (ERI)	1:03:07	2:12:03
Lemoncello, Andrew (GBR)	1:05:27	2:13:40
Kifle, Yonas (ERI)	1:03:06	2:14:39
Jones, Andi (GBR)	1:05:45	2:16:38

40 to 44 years

Bilton, Darran E (GBR)	1:11:24	2:25:08
Cope, Nicholas (GBR)	1:10:33	2:26:18
Vautier, Stephane (FRA)	1:11:56	2:27:53
Weir, Andrew P (GBR)	1:14:04	2:29:30
Lincoln, Wayne (GBR)	1:14:01	2:30:08
Richmond, Kenny (GBR)	1:14:14	2:31:41
Smalls, Allen (GBR)	1:16:36	2:33:23
Berg, Jens – Kristian (NOR)	1:15:10	2:34:35
Knight, Geoffrey P (GBR)	1:15:29	2:35:14
Freeman, David J (GBR)	1:17:07	2:35:56

45 to 49 years

Crampton, Ian (GBR)	1:12:58	2:30:02
Larripa, Eric (FRA)	1:15:17	2:33:02
Rackham, Nigel D (GBR)	1:17:28	2:36:05
Downs, Rob H (GBR)	1:15:46	2:37:33
Hatton, Michael (GBR)	1:18:39	2:38:58
Foster, Philip A (GBR)	1:17:08	2:39:42
Waby, Paul (GBR)	1:18:38	2:39:43
Gee, Sarah R (GBR)	1:18:35	2:40:06
Reilly, Leonard J (GBR)	1:20:43	2:40:15
Croasdale, Mark J (GBR)	1:15:16	2:40:21

Data from http://results-2010.virginlondonmarathon.com/2010/

Work out the mean time in each age group that these runners took to:

1 Get half way round the marathon course,
2 Complete the marathon.

Work out the range of times in each age group that these runners took to:

1 Get half way round the marathon course,
2 Complete the marathon.

Are older runners slower than younger runners?

You may use this further data from the spreadsheet and website.
Show how you decide.

Averages (London Marathon) - Level 2

Introduction

The aim of the activity is to give learners practise in extracting information from tables, calculating means and range and use these to answer a simple question.
The task is Level 2 and is difficult for as it involves continuous data and time.

The task could be introduced to the whole group and issues discussed. No questions have been posed so learners need to decide a strategy around means and ranges and making comparisons using these.

Many learners find the process of calculating means relatively easy (and mechanical) but cannot interpret the results. They will need to express their ideas to the group (or teacher) and be corrected on poor use of the measures of location.
Ideas could be discussed about the limited amount of data available and how this could be remedied.

This task is ideal for ICT and the spreadsheets may be used and added to.

Relevant figures for the sets of data are:

18 to 39 years	Mean (half way)	1h 3m 37s	Range	2m 39s
	Mean (complete)	2h 10m 0s	Range	11m 19s
40 to 44 years	Mean (half way)	1h 14m 3s	Range	6m 34s
	Mean (complete)	2h 30m 59s	Range	10m 48s
45 to 49 years	Mean (half way)	1h 17m 3s	Range	7m 45s
	Mean (complete)	2h 37m 35s	Range	10m 19s
50 to 54 years	Mean (half way)	1h 20m 2s	Range	7m 21s
	Mean (complete)	2h 44m 13s	Range	14m 52s

Learners could work in pairs if ICT is not used or individually to process data on spreadsheets.
Further data may be downloaded from this website.

http://results-2010.virginlondonmarathon.com/2010/

The data will need to be downloaded sheet by sheet and pasted into the spreadsheet.

It is important that outcomes are discussed and learners justify their assertions with use of data.

Functional Standards

R1	Understand that means give representative values for groups of runners, decide to make use of these and also refer to tables to compare individual runners
R2	Extract **relevant** information from the tables
R3	Know correct methods to calculate mean and range and decide how to use these results and individual results to answer the question, "Are older runners slower than younger ones?"
A1	Use correct methods to calculate mean and range as well as compare relevant individual runner's results.
A2	Repeat calculations or apply some checking method or check that results seem sensible.
I1 I2	Answer the question "Are older runners slower than younger ones?" with more than a "Yes/No" answer. Use their results and explain how they inform their answer. Make some comment on such things as accuracy, limited data etc.

Key Stage 4 references (J562)

FA13 General data handling	
13.1 - Understand and use statistical problem solving process/handling data cycle	Candidates should be able to: a) carry out each of the **four** aspects of the handling data cycle to solve problems: i) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources; ii) process and represent the data: turn the raw data into usable information that gives insight into the problem; b) Interpret and discuss the data: answer the initial question by drawing conclusions from the data.
13.2 - Experimenting	Candidates should be able to: a) discuss how data relate to a problem, identify possible sources of bias and plan to minimise it; b) identify key questions that can be addressed by statistical methods; c) design an experiment or survey and decide what primary and secondary data to use; d) design and use data-collection sheets for grouped discrete and continuous data; e) gather data from secondary sources, including printed tables and lists from ICT-based sources;
13.3 - Processing	Candidates should be able to: a) draw and produce pie charts for categorical data, and diagrams for continuous data, frequency diagrams (bar charts, frequency polygons and fixed interval histograms) and stem and leaf diagrams; b) calculate mean, range and median of small data sets with discrete then continuous data;

13.4 - Interpreting	Candidates should be able to: a) look at data to find patterns and exceptions; b) interpret a wide range of graphs and diagrams and draw conclusions; c) interpret social statistics including index numbers; and survey data; d) compare distributions and make inferences, using the shapes of distributions and measures of average and range; e) understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better population characteristics.

Resources Needed

Calculators, computer terminals

Misconceptions and common errors

- Candidates frequently fail to take note of headings in tables.
- They transcribe data from the tables incorrectly.
- Candidates often do not show the working they have used to obtain answers and so it is often difficult to know whether they have used correct figures or not.
- Answers are often poorly expressed. Learners often write at length but fail to communicate ideas clearly. Simple notes and headings are often preferable to prose
- Candidates can apply mechanical processes but often do not understand the outcome. Means may be calculated but their significance as a representative value is often poorly understood.

Previously set Functional Mathematics questions

Abasi and Owen are talking about the countries that won medals in the 2006 Winter Paralympics in Turin.

(A table of data was supplied)

They collect information about some countries that won medals.

The information that Abasi and Owen collected is shown in the Resource Booklet.

Use the information in the table to investigate whether:

(a) wealthy countries won more gold medals,

(b) big countries won more gold medals.

You must show your calculations. State how you have used them and any charts you might draw to get your answer.

The Principal Examiner commented:

Most candidates, who attempted this task, drew graphs of some form or another. Only a very small number used averages and these were to support their graphical approaches. Some understood that they were to explore the connection (or not) between the three fields - country, number of gold medals and the GDP. A small number tried to categorise countries into rich/poor or big/small by looking for natural break-points this often split the countries into 4 and 11.

Unfortunately there were only a small number of scatter graphs drawn, but these tended to get almost full credit. However, those who drew bar or line graphs merely drew the countries in the order they were in the resource booklet and gained minimal credit.

Main Activity: Averages (London Marathon) – Level 2

London Marathon 2010

Here is the data for the first 10 finishers in the London Marathon 2010 for four different age groups.

18 to 39 years

Name	Half time (h : m : s)	Finish time (h : m : s)
Kebede, Tsegaye (ETH)	1:03:07	2:05:19
Mutai, Emmanuel (KEN)	1:03:06	2:06:23
Gharib, Jaouad (MAR)	1:03:07	2:06:55
Bouramdane, Abderrahime (MAR)	1:03:07	2:07:33
Kirui, Abel (KEN)	1:03:06	2:08:04
Gomes dos Santos, Marilson (BRA)	1:03:07	2:08:46
Tadese, Zersenay (ERI)	1:03:07	2:12:03
Lemoncello, Andrew (GBR)	1:05:27	2:13:40
Kifle, Yonas (ERI)	1:03:06	2:14:39
Jones, Andi (GBR)	1:05:45	2:16:38

40 to 44 years

Bilton, Darran E (GBR)	1:11:24	2:25:08
Cope, Nicholas (GBR)	1:10:33	2:26:18
Vautier, Stephane (FRA)	1:11:56	2:27:53
Weir, Andrew P (GBR)	1:14:04	2:29:30
Lincoln, Wayne (GBR)	1:14:01	2:30:08
Richmond, Kenny (GBR)	1:14:14	2:31:41
Smalls, Allen (GBR)	1:16:36	2:33:23
Berg, Jens – Kristian (NOR)	1:15:10	2:34:35
Knight, Geoffrey P (GBR)	1:15:29	2:35:14
Freeman, David J (GBR)	1:17:07	2:35:56

45 to 49 years

Crampton, Ian (GBR)	1:12:58	2:30:02
Larripa, Eric (FRA)	1:15:17	2:33:02
Rackham, Nigel D (GBR)	1:17:28	2:36:05
Downs, Rob H (GBR)	1:15:46	2:37:33
Hatton, Michael (GBR)	1:18:39	2:38:58
Foster, Philip A (GBR)	1:17:08	2:39:42
Waby, Paul (GBR)	1:18:38	2:39:43
Gee, Sarah R (GBR)	1:18:35	2:40:06
Reilly, Leonard J (GBR)	1:20:43	2:40:15
Croasdale, Mark J (GBR)	1:15:16	2:40:21

50 to 54 years

Clements, Andrew C (GBR)	1:15:15	2:33:14
Payne, Garry P (GBR)	1:17:24	2:39:32
Szwinto, Henry E (GBR)	1:19:32	2:41:11
Hetherington, Gary (GBR)	1:17:59	2:45:05
Clarke, David R (GBR)	1:22:36	2:45:45
Smythe, Stephen J (GBR)	1:21:28	2:46:06
Stephens, Graham C (GBR)	1:22:07	2:47:27
Burton, Robert (GBR)	1:22:50	2:48:04
Teece, Philip R (GBR)	1:21:58	2:48:06
Kitching, Ian D (GBR)	1:19:07	2:47:37

Data from http://results-2010.virginlondonmarathon.com/2010/

Are older runners slower than younger runners?

You may use this data and, if you wish, further data from the spreadsheet and website.
Show how you decide.

Starter Activity: Averages - Level 2 Notes

Marathon

This is a short starter that encourages learners to extract and interpret data from graphs.
A time should be set (say five minutes) and learners work, on their own, to write down their findings.
Discussion should take place afterwards when learners present and justify their answers.
A projection of the graph is essential and learners should be encouraged to stand before the group and present their findings.

Many statements could be made but probe general statements like “People are quicker” with a, “How do you know?” follow up.

- Scales cannot be read accurately.
- For both genders the trend is downwards.
- Women did not record times (didn’t compete) before the 1970s.
- The rate of improvement for women is more rapid than that for men.
- Few records are set at Olympic games.
- Women, on average, run the marathon more slowly than men taking roughly 12 minutes longer.
- The average speed of a marathon runner is just under 13 miles per hour.
- Etc

Starter Activity: Averages - Level 2

London Marathon 2010

This information is about the top ten finishers in the 2010 London Marathon.

Place Overall	Name	Age (years)	Half time (h : m : s)	Finish time (h : m : s)
1	Kebede, Tsegaye (ETH)	18-39	01:03:07	02:05:19
2	Mutai, Emmanuel (KEN)	18-39	01:03:06	02:06:23
3	Gharib, Jaouad (MAR)	18-39	01:03:07	02:06:55
4	Bouramdane, Abderrahime (MAR)	18-39	01:03:07	02:07:33
5	Kirui, Abel (KEN)	18-39	01:03:06	02:08:04
6	Gomes dos Santos, Marilson (BRA)	18-39	01:03:07	02:08:46
7	Tadese, Zersenay (ERI)	18-39	01:03:07	02:12:03
8	Lemoncello, Andrew (GBR)	18-39	01:05:27	02:13:40
9	Kifle, Yonas (ERI)	18-39	01:03:06	02:14:39
10	Jones, Andi (GBR)	18-39	01:05:45	02:16:38

Data from http://results-2010.virginlondonmarathon.com/2010/

1 Were these runners in the same order half way through the course?
2 After Kebede won the race, for how long did Jones have to run to finish the race?
3 How old was Mutai when he came second?

The mean finish time for the first seven women was 2 hours 44 minutes and 11 seconds.

4 How much faster, on average, were the first seven men than the first seven women?

These are the record times for marathon races

Men

Rank	Time	Name	Country	Race	Place	Race Date
1	2:03:59	Gebrselassie, Haile	ETH	Berlin	1	9/28/08

Women

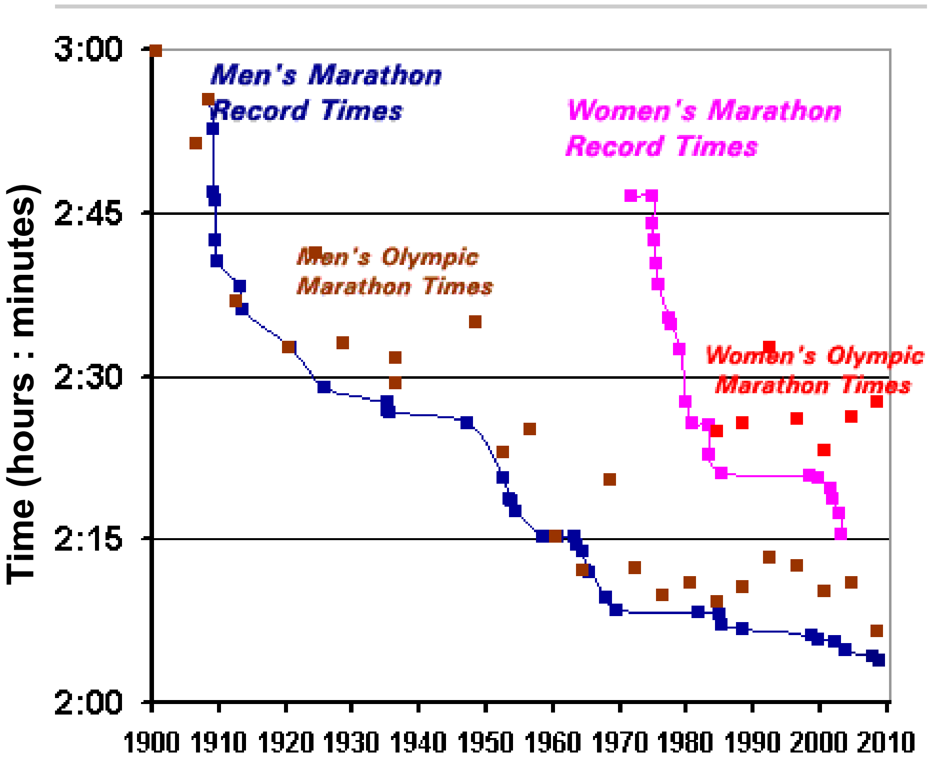
Rank	Time	Name	Country	Race	Place	Race Date
1	2:15:25	Radcliffe, Paula	GBR	London	1	4/13/03

Data from http://www.marathonguide.com

5 In what position might Paula Radcliffe have finished in the London Marathon 2010 if she had been running?
6 Explain your answer.

Marathon

This graph shows the way that the record times for the marathon have changed.



Write down as many things as you can that the graph tells you about these marathon record times.

The marathon is run over a distance of 26 miles.

You have 5 minutes.

Functional Skills Maths

Functional skills qualification in Maths at Level 1 and Level 2

Number

General Information

Introduction

One of the key elements of being mathematically functional is the ability to make comparisons: seeing a link between two or more quantities and using that link to determine a missing quantity. (Note, however, that the types of comparison may vary depending upon the question or context). All of the following topics involve ratio in one way or another: changing between units of measurement; distance/speed/time; exchange rates; recipes and mixtures, map scales and scale drawing.

GCSE Descriptors and Mathematics Functional Skills Criteria

GCSE Descriptors relating to ratio Foundation Tier.

FA4 Ratio; HA4 Ratio	
4.1 - Use ratio notation, including reduction to its simplest form and its various links to fraction notation	Candidates should be able to: a. use ratio notation, including reduction to its simplest form; b. know its various links to fraction notation.
4.2 - Divide a quantity in a given ratio	Candidates should be able to: a. divide a quantity in a given ratio; b. determine the original quantity by knowing the size of one part of the divided quantity; c. solve word problems about ratio, including using informal strategies and the unitary method of solution.

Functional Skills Mathematics Qualification Criteria

Level	Coverage and range with links to ratio
Entry 1	▪ describe the properties of size and measure, including length, width, height and weight, and make simple comparisons.
Entry 2	▪ Use doubling and halving in practical situations ▪ Use simple scales and measure to the nearest labelled division
Entry 3	▪ Understand, estimate, measure and compare length, capacity, weight and temperature
Level 1	▪ Solve simple problems involving ratio, where one number is a multiple of the other.
Level 2	▪ Understand, use and calculate ratio and proportion, including problems involving scale

Note: the examiner comments raise some teaching points.

Level 1

Questions involving ratio tend to be fairly well answered at this level, as the following slightly adapted question will illustrate:

> Jade picks 10 kg of damsons from her garden and decides to make jam to sell at her local market.
> Jade's recipe for damson jam says that every 2.5 kg of damsons needs to be mixed with 3 kg of sugar and 1 litre of water and then boiled for half an hour.
>
> (a) How much sugar does she need to mix with 10 kg of damsons?
> [4 marks]

The examiner's report noted:
"This was often correctly answered although a common error was 2.5(kg) x 10."
The reasons for the error may be a failure to identify the appropriate information or to "just work with damsons" and ignore the stated ratio of sugar to damsons.

The mark scheme for this part of the question was:

1	Valid attempt to find sugar for one other quantity of damsons [eg 5 kg of damsons needs 6 kg sugar] OR Statement 2.5 kg needed for 3 kg damsons
2	**Find multiplier [10 ÷ 2.5] = 4 MAY BE IMPLIED BY CORRECT ANSWER IN RF**
1	12 or 12 kg (no evidence) OR 12 with evidence
2	**Full proportional method AND 12 kg**

Level 2:

The following two tasks are included in some detail. You may wish to try them, either as presented or adapted, with some students who are working at level 2.

Question 1 – Fruit Drinks

Hannah has been asked to produce a fruit drink for a summer fete.

She decides to experiment with a drink made from cranberry juice and sparkling (fizzy) mineral water.

For her experiments, she tries these four different recipes:
(these were presented in the Resource Book and presented below)

Recipe A	**Recipe B**
2 measures of cranberry juice 3 measures of mineral water	4 measures of cranberry juice 8 measures of mineral water
Recipe C	**Recipe D**
3 measures of cranberry juice 5 measures of mineral water	1 measure of cranberry juice 4 measures of mineral water

She tried the recipes on some friends and almost everybody preferred the strongest cranberry flavour.
She decides to use this recipe.

(a) Which recipe, A, B, C or D, did Hannah decide to use?

Show how your working leads to your answer.

[5 marks]

The mark scheme for this question was:

Comparing strengths of recipes **[A]**	**1** **1** **1** **1** **1**	2:3 is equivalent to 1:1.5 or $\frac{2}{5}$ or equivalent seen 4:8 is equivalent to 1:2 or $\frac{4}{12}$ or equivalent seen 3:5 is equivalent to 1:0.375 or $\frac{3}{8}$ or equivalent seen 1:4 is equivalent to 1:4 or $\frac{1}{5}$ or equivalent seen For correct answer: " A is the stronger." If zero scored above – lack/incorrect working and correct answer award SC2 Award 4 for A selected and sight of 2:3 o.e. in context of A stated as being the strongest. E.g. "A is the strongest because it has (2:3, 2 parts of 5 etc.) of cranberry."

The examiner's report noted:
"A significant proportion had an intuitive understanding of the effect of the numbers on the strength without being able to show it mathematically. .... A number explained how they eliminated the weakest recipes and then had problems differentiating between A & C. The mathematical relationship between ratios and fractions was in general misunderstood by many candidates and a less than secure understanding of ratio clearly evident. There were some who just chose recipe A because there was only one difference between the cranberry juice and water. "

Question 2 – Muesli

(note this question used extensive information which was presented in the Resource Booklet. This information is given below.

Paul likes muesli. He wants to make his own from a recipe he found on the Internet.

Paul looks at prices in his local health food shop and chooses hazelnuts and apricots.

(a) Paul buys Nature's Bounty muesli. It costs £1.89 for a 500g bag.
Can Paul save money by making his own muesli?
Clearly show how your working leads to your answer.
[11 marks]

(b) In the Resource Booklet, there are some nutritional facts about Nature's Bounty muesli and information about sugars in foods.
Which muesli has the least amount of sugars, Paul's or Nature's Bounty?
Clearly show how your working leads to your answer.
[7 marks]

The following 2 panels give the Information for Muesli which was extracted from the Resource Booklet and reproduced here.

(a) Paul found this muesli recipe on the Internet.

Muesli
40% oats
40% wheat flakes
10% nuts
10% dried fruit

Here is part of the price list in his local health food shop.

Each Price is for one kilogram	
Oats	**£1.60**
Wheat flakes	**£1.60**
Dried Fruit	
Apple slices	**£19.20**
Apricots	**£7.80**
Banana chips	**£4.70**
Nuts	
Almonds	**£22.40**
Brazil Nuts	**£14.90**
Hazelnuts	**£11.90**

(a) Here are some nutritional facts about Nature's Bounty muesli.

Nature's Bounty Muesli
Nutritional Facts

Serving size (50 g)

Total fat	**4 g**
Sugars	**12 g**
Salt	**75 mg**

The health food shop has a leaflet about sugars in foods.

Weight of sugars per 100 g

	Oats	0 g
	Wheat flakes	0 g
Dried fruit		
	Apple slices	55 g
	Apricots	90 g
	Banana chips	65 g
Nuts		
	Almonds	4 g
	Brazil nuts	4 g
	Hazelnuts	5 g

The mark scheme for this question was:

Part(a)		
Finding relative weights of ingredients [A]	2 1	Oats: 200g Wheat flakes: 200g Hazelnuts: 50g Dried apricots: 50g or equivalents 2 or 3 correct A2 or A1 may be implied from [B].
Finding costs of ingredients (for 500g or [1 kg]) [B]	6	M1 + A1 or B2 for each set – maximum of 6 Oats: $\left(\frac{"200"}{1000} \text{ or } 0.2 \text{ or equivalent}\right) \times £1.60 \rightarrow 32\text{p } [64\text{p}]$ Wheat flakes: $\left(\frac{"200"}{1000} \text{ or } 0.2 \text{ or equivalent}\right) \times £1.60 \rightarrow 32\text{p } [64\text{p}]$ Hazelnuts: $\left(\frac{"50"}{1000} \text{ or } 0.05 \text{ or equivalent}\right) \times £11.90 \rightarrow 59.5\text{p}$ [allow 59p or 60p – or for 1 kg 1.19/119]

		Apricots: $\left(\frac{"50"}{1000} \text{ or 0.05 or equivalent}\right) \times £7.80 \rightarrow 39p$ [78p]
Calculating total cost of 500g of homemade muesli [C]	**1**	£1.62 or £1.63 or allow full (correct) follow through from their sub-totals
Conclusion on relative costs [D]	**2** **1**	Paul's muesli is cheaper by 26p / 27p / [53p] correct conclusion based on candidate's figures.
Part(b)		
Calculating sugars in a "portion" of Paul's muesli.		*Based on Paul's original 500g of muesli – but other equivalent approaches are possible.*
Finding sugar in oats and wheat flakes [E]	**1**	Oats and wheat flakes zero sugar or equivalent – must have specific statement – not by default or implication.
Finding sugar in hazelnuts in Paul's muesli [F]	**1**	In 50 g of hazelnuts there are 2.5 g of sugar or correct mass in any other mass apart from 100g but allow if it is clear that the candidates is dealing with 100g as evidenced from [A] or [B].
Finding sugar in apricots in Paul's muesli [G]	**1**	In 50 g of apricots there are 45 g of sugar or correct mass in any other mass apart from 100g but allow if it is clear that the candidates is dealing with 100g as evidenced from [A] or [B].
Calculating total sugar in 500g of Paul's muesli. [H]	**1**	2.5 + 45 = 47.5g, full following through from above calculations. H0 if E0 (to stop incorrect comparisons).
Comparing Paul's muesli with the same weight of muesli Nature's Bounty muesli [I]	**1**	In 50 g portion of Paul's there is 4.75g of sugar, follow through on candidates' correct figures.
Statement comparing Paul's with supermarket [J]	**1**	Statement comparing Paul's with supermarket based on the candidates' shown working.
Checking calculations [K]	**1**	Evidence of checking by re-working or commenting on reasonableness of results at any point in task or no obvious purely arithmetical error – at least three pure arithmetic calculations without error or mention of reasonableness of results at any point in task.

The examiner's report stated:

"A common error was to assume that Paul chose the cheapest ingredients initially, a result perhaps of not reading the second line of text. Several valid methods were used by candidates to compare prices. The most popular was to compare the prices of one kilogram of muesli. Those who interpreted the situation and mathematics required to solve the first problem tended to do well. Some of the more obviously incorrect answers might not have been submitted had candidates reflected more on them. A majority of candidates were able to gain half or more of the available marks for the first part of the task.

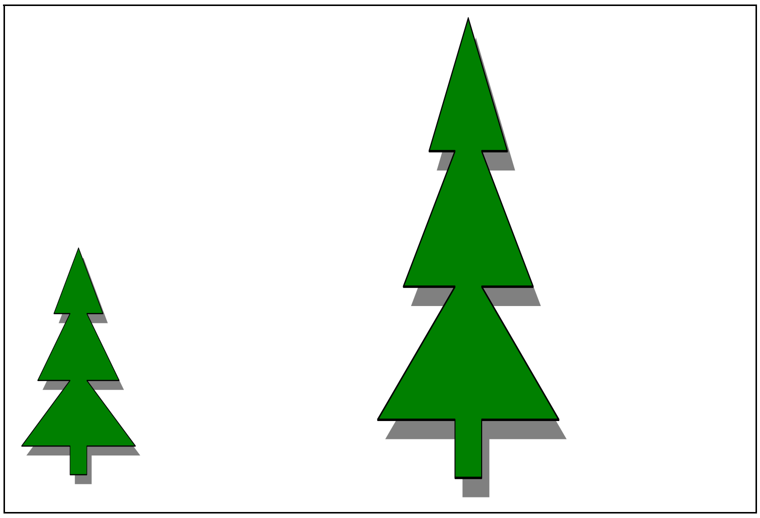
The second part was found more challenging by many candidates who found working with ratio in this practical situation challenging. It was not uncommon for the total mass sugars present in a portion to be considerably greater than the 100g for a 50g or 100g portion of muesli itself. This reinforces the comment about candidates looking and considering answers from the point of view of being reasonable and asking themselves "is this answer sensible?"."

Common errors and misconceptions

Ratio and fractions: although often seen as different topics in a mathematics syllabus are linked – e.g. a ratio is given as 2:3 or 3:5. and it seems an obvious question to ask what fraction each part is of the whole. Thus: Gemma is sharing some sweets with Garry. Gemma says “two for you and three for me. Two for you and three for me ....” until all the sweets have gone. Mary watches Gemma and when all the sweets have been shared out Mary says “ Gemma has got $\frac{2}{3}$ of the sweets” Here, of course, Mary is confusing the ratio 2:3 which is comparing part with part, and the fraction $\frac{2}{3}$ which is comparing part with whole. The confusion is between “2 for every 3” and “2 out of every three”

In this drawing the outline of the small tree has a height of 2cm and the outline for the large tree has a height of 5cm.

The large tree’s real height is 30cm. What is the real height of the small tree?



A question like this can cause some confusion and illustrate some misconceptions:
Solutions are:

$\frac{2}{5} = \frac{x}{30}$ giving $x = 12\text{cm}$

or 5 x ? = 30 so ? = 6 Therefore 2 x 6 = 12cm
An incorrect solution could be 27cm from using an additive strategy thus 5 +25 = 30 so 2 + 25 = 27.

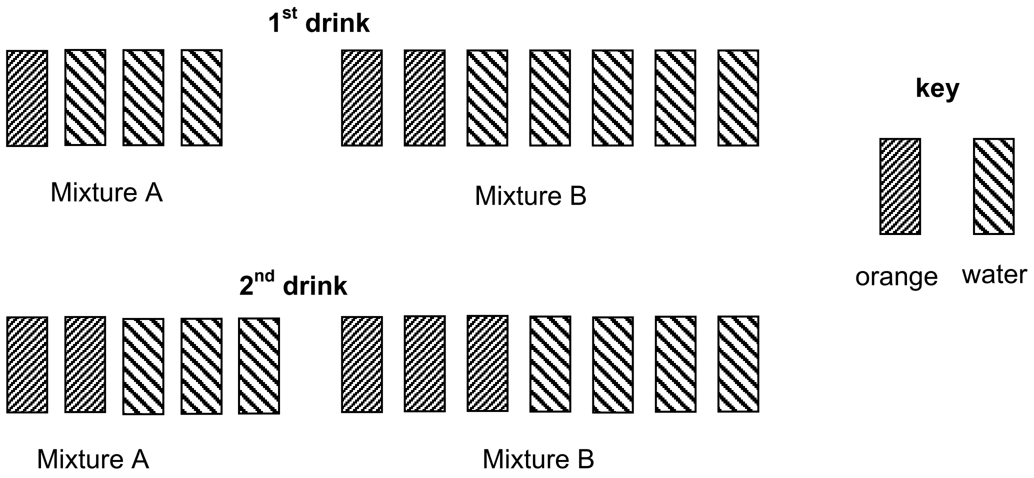
Similar addition type misconceptions occur with recipe problems. For example

To cook a meal for 12 people I need:
- 36 sausages
- 6 tomatoes
- 18 potatoes
- 1kg of peas
- 1kg of carrots

how many potatoes will I need to cook a meal for 8 people?

The solution expected would be 12 potatoes because the ratio of people to potatoes is 2:3 but a common misconception is to think the number of potatoes is 6 more than the number of people and give the answer 14.

A common problem in assessments, (see the Level 2 question above), involves the "oranginess" of drinks, for example:



There are 2 basic strategies to establish which is the "orangiest" drink – 'between' and 'within'.

For the 1st drink

- using the 'between' strategy: for 1 glass of juice there are three glasses of water. Therefore with 2 glasses of juice there should be 6 glasses of water but in B there are only 5 glasses of water, therefore B is orangier.
- Using the 'within' strategy:: there is twice as much juice in B as in A but there is not twice as much water, therefore B is orangier

For the 2nd drink

- Using the 'between' strategy: the amount of juice in B is $\frac{3}{2}$ times that in A but $\frac{3}{2}$ times as much water would make $\frac{9}{2}$ glasses of water in B but there are only 4 glasses of water in B therefore B is orangier
- Using the 'within' strategy: the ratio of juice to water in A is 2: 3 or the fraction of juice is $\frac{2}{5}$ while in B the ratio of juice to water is 3:4 or the fraction of juice is $\frac{3}{7}$. Comparing $\frac{2}{5}$ and $\frac{3}{7}$, ie $\frac{14}{35}$ with $\frac{15}{35}$ then B is orangier.

The activities

The following activities are designed to give opportunities for students to work with simple ratios through analysing and making patterns according to a set of rules or instructions. They are intended to offer a different approach to the usual textbook and examination type questions.

Patterns using counters

Starter Activity 1

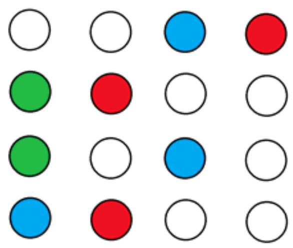
Target Group	Y8 to Y10
Introduction	This can be set for students to work individually or in pairs.
Underpinning knowledge	Knowledge of ratio
Materials	Photocopied sheet.. 6 red, 6 blue and 6 green counters. (or coloured pencils)
Functional skills likely to be used	At level 1 – solve simple problems involving ratio At level 2 – understand and use ratio.
Guidance and notes	The activity should take no more than 5 minutes. Level 1 students could be given the partially completed pattern as shown below and asked to finish it according to the clues provided. Level 2 students could be given a blank 4 by 4 “grid” and 6 red, 6 blue and 6 green counters and asked to make the complete pattern using the clues provided.

Worksheets provided.
Level 1: complete the pattern using the clues provided.
Level 2: Make the pattern following the clues provided.

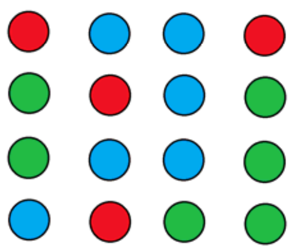
Clues:
Ratio of red : blue = 2:3
Ratio of green : red = 3:2

Columns
1: Red : green = 1:2
3: ¾ are blue
4: Red : green = 1:3

Rows
1: ½ are red
2: Green : red = 2:1
4: ½ are green



Solution:



Starter Activity 2

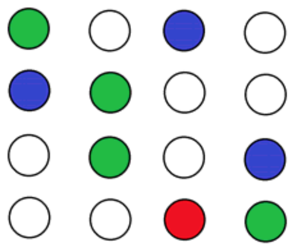
Target Group	Y8 to Y10
Introduction	This is a harder activity which can be set for students to work individually or in pairs, ideally pairs.
Underpinning knowledge	Knowledge of ratio
Materials	Photocopied sheet. 8 red, 8 blue and 8 green counters. (or coloured pencils)
Functional skills likely to be used	At level 1 – solve simple problems involving ratio At level 2 – understand and use ratio.
Guidance and notes	The activity should take no more than 5 minutes. Level 1 students could be given the partially completed pattern as shown below and asked to finish it according to the clues provided. Level 2 students could be given a blank 4 by 4 "grid" and 8red, 8 blue and 8 green counters and asked to make the complete pattern using the clues provided.

Level 1: complete the pattern using the clues provided.
Level 2: Make the pattern following the clues provided.

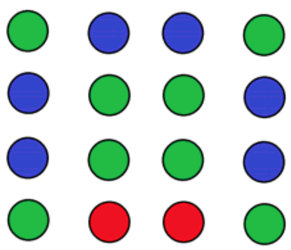
Complete the pattern using the clues provided
Clues
Ratio of green : blue = 4:3
Ratio of red : blue = 1:3

Columns
1: Blue : green = 1:1
2: Red : green = 1:2
3: Red : green = 1:2

Rows
1: ½ are blue
2: Green : blue = 1 : 1
4: ½ are green



Solution



Starter Activity 3

Target Group	Y8 to Y10
Introduction	This could be a homework challenge.
Underpinning knowledge	Knowledge of ratio
Task	Ask students to make up their own pattern– with just 3 column and row clues – Challenge question: what is the smallest number of given colours to solve?

Main activity 1

Target Group	Y8 to Y10
Time	Approximately 20 minutes
Introduction	This is a longer activity which builds on the work of the starter activity 1
Mode of working	Groups of 4 or 5 students
Underpinning knowledge	Knowledge of ratio
Materials	Playing Boards/activity sheets – 1 per group. Set of 18 cards per group and 20 red, 20 blue and 20 yellow counters per group
Possible content	Development of ratio and symmetry
Functional skills likely to be used	At level 1 – solve simple problems involving ratio At level 2 – understand and use ratio.
Instructions and notes	The activity follows on from the starter activity. The aim is to arrange counters on the board so that the pattern matches the clues on all the cards. Deal out all the cards between the players. Each person looks at their own cards and can read out the information on their cards at any time and as often as is required. The group should work together to decide where the counters should go. All the counters will not be used.

Main activity 2

Target Group	Y8 to Y10
Time	Approximately 20 minutes
Introduction and notes	This builds from activity 4. Prompts and questions to be asked could be: 1 what is the smallest number of clues you need to solve the problem? 2 Design a different pattern on the board. Make up a set of clues for your pattern and give them to another group to solve
Development	Adding more columns and rows gives possibilities for a wider varieties of ratios and further use of symmetry

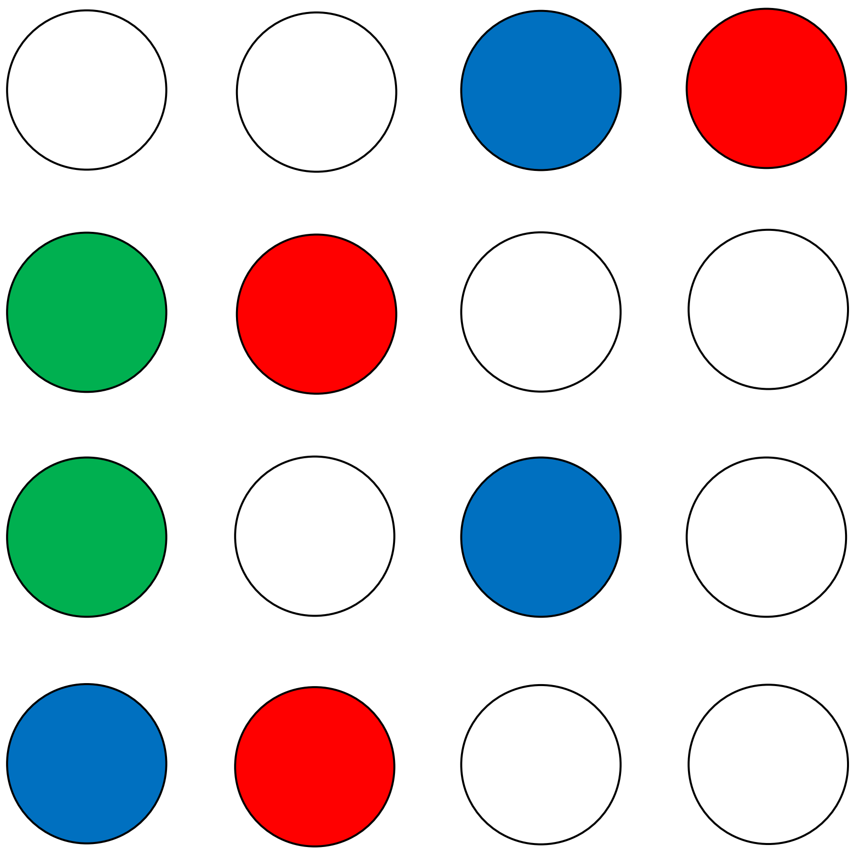
Starter Activity 1 (Level 1)

Complete the pattern using the clues provided.

Clues:
Ratio of red : blue = 2:3
Ratio of green : red = 3:2

Columns
1: Red : green = 1:2
3: ¾ are blue
4: Red : green = 1:3

Rows
1: ½ are red
2: Green : red = 2 : 1
4: ½ are green



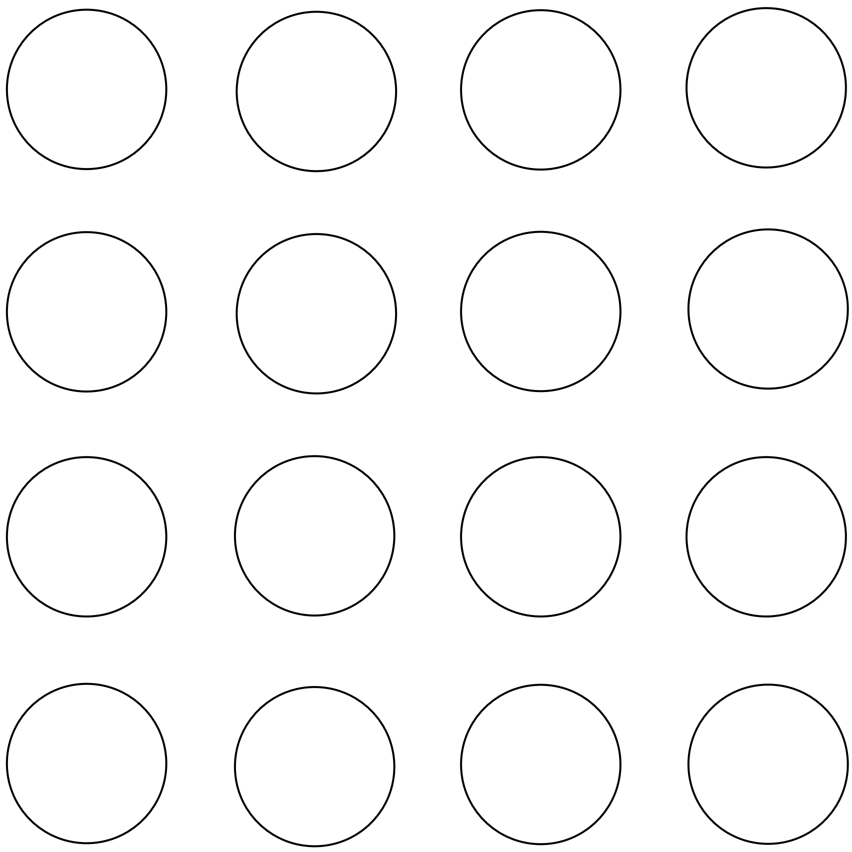
Starter Activity 1 (Level 2)

Make the pattern following the clues provided.

Clues:
Ratio of red : blue = 2:3
Ratio of green : red = 3:2

Columns
1: Red : green = 1:2
3: ¾ are blue
4: Red : green = 1:3

Rows
1: ½ are red
2: Green : red = 2 : 1
4: ½ are green



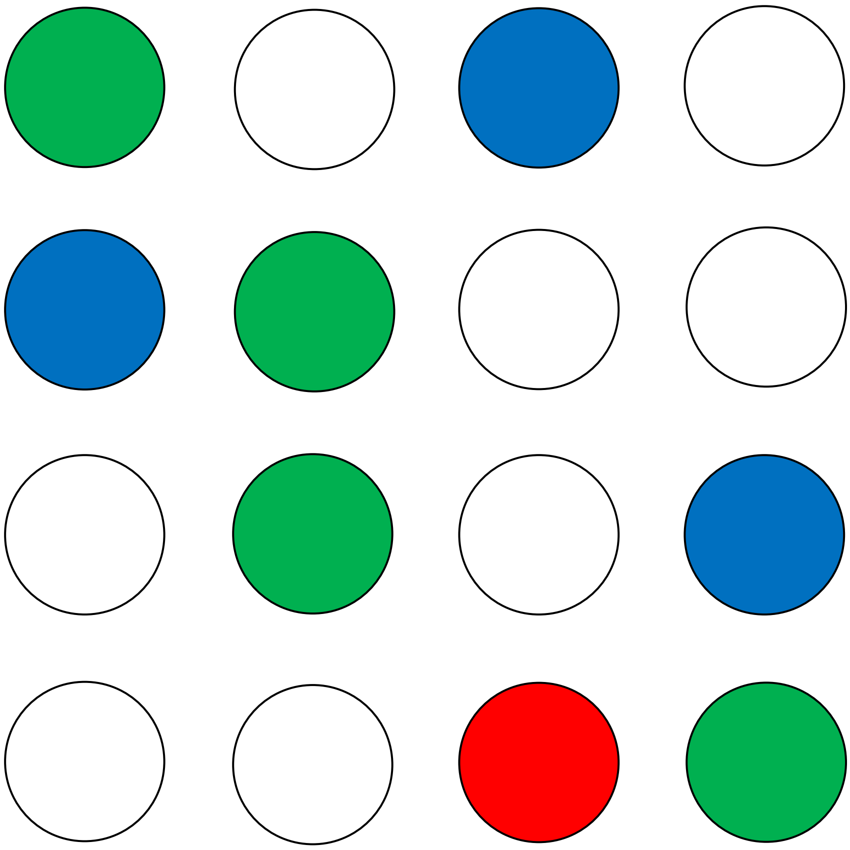
Starter Activity 2 (Level 1)

Complete the pattern using the clues provided.

Clues
Ratio of green : blue = 4:3
Ratio of red : blue = 1:3

Columns
1: Blue : green = 1:1
2: Red : green = 1:2
3: Red : green = 1:2

Rows
1: ½ are blue
2: Green : blue = 1 : 1
4: ½ are green



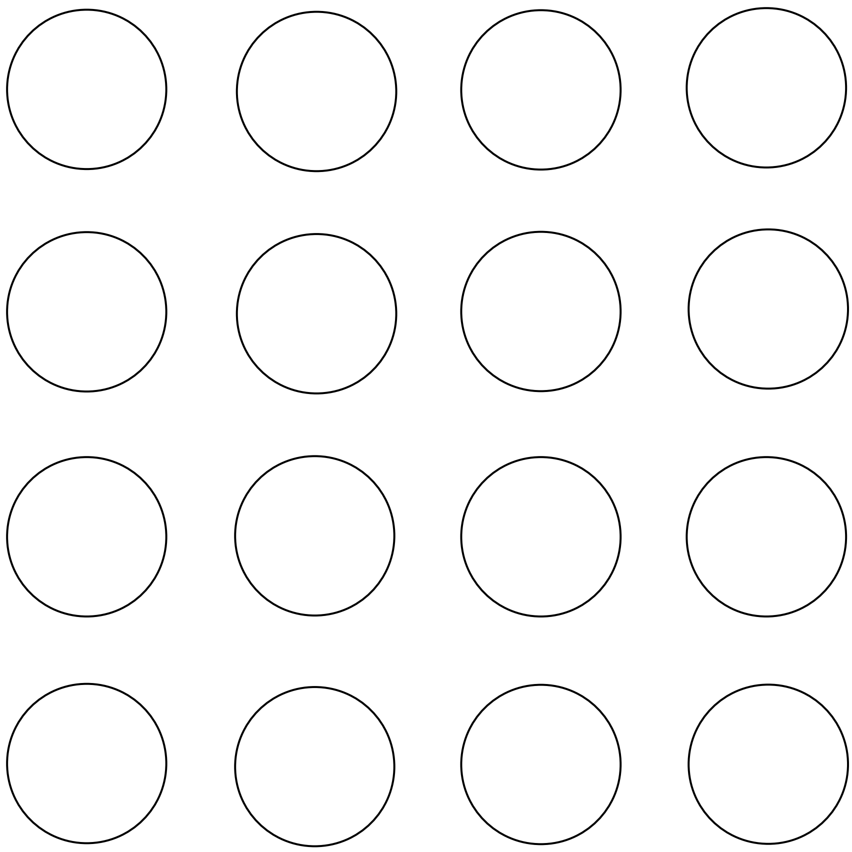
Starter Activity 2 (Level 2)

Make the pattern following the clues provided.

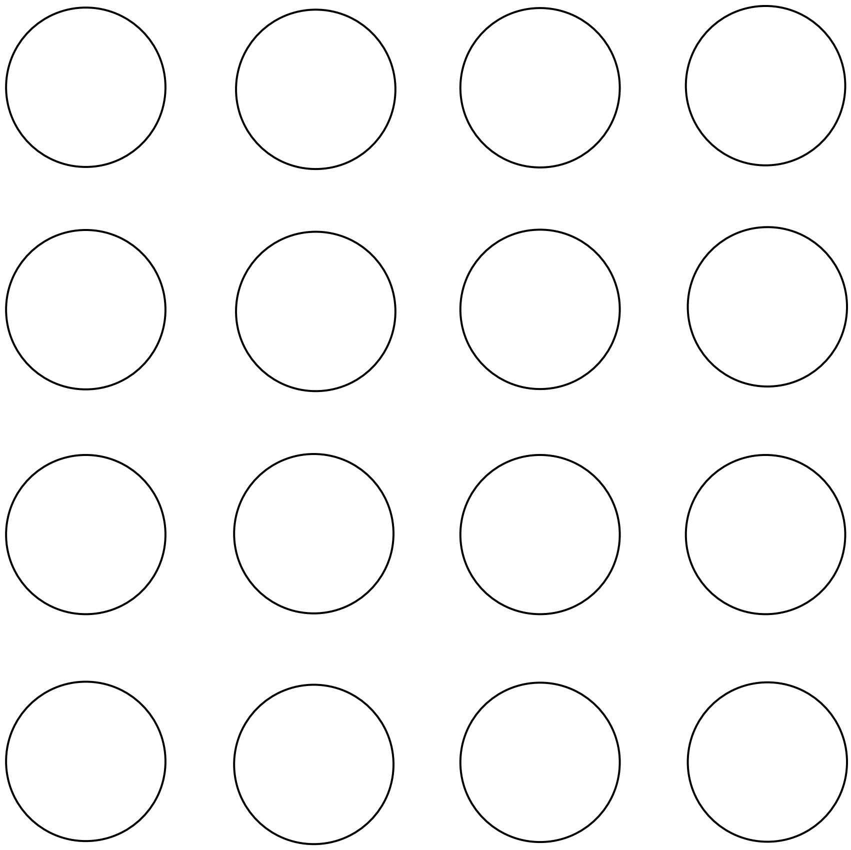
Clues
Ratio of green : blue = 4:3
Ratio of red : blue = 1:3

Columns
1: Blue : green = 1:1
2: Red : green = 1:2
3: Red : green = 1:2

Rows
1: ½ are blue
2: Green : blue = 1 : 1
4: ½ are green

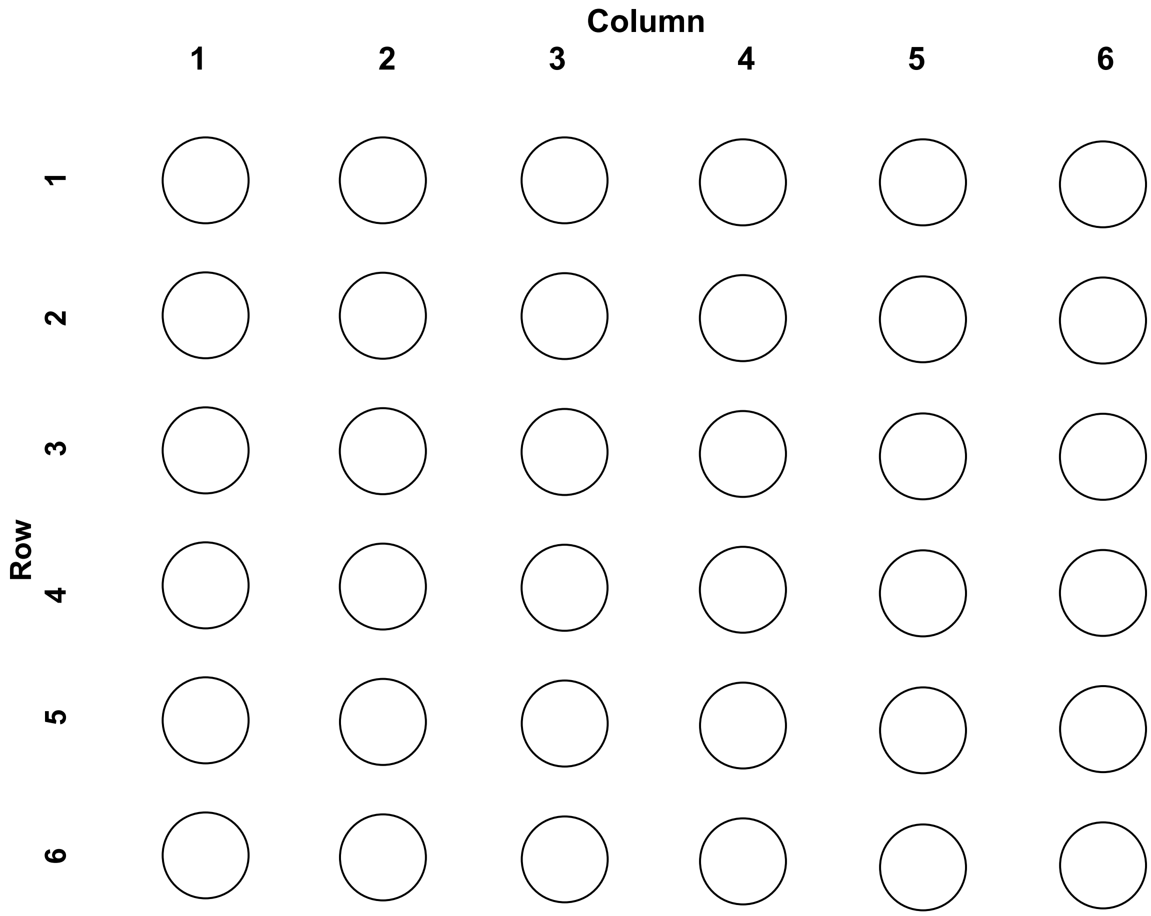


Starter Activity 3



Main Activity 1

There are twice as many blue counters as red counters	The ratio of blue counters to yellow counters = 4 : 3
Rows 1 and 6 are the same	Columns 3 and 4 are the same
Row 2 has 2 red counters	Row 4 has twice as many blue counters as yellow counters
The ratio of red to yellow counters is 2 : 3	One third of the counters are yellow
Column 5 has 2 yellow counters	There are the same number of red and blue counters in column 5
All four corners are the same colour	Rows 3 and 4 have no red counters
Column 4 has no red counters	The counter in position (2, 2) is red
The complete pattern looks symmetrical	The diagonal from top left to bottom right has no blue counters
Row 2 has 2 blue counters	The counter which is in column 1, row 1 is red


Column
1
2
3
4
5
6
Row
1
2
3
4
5
6

Ellen's bedroom

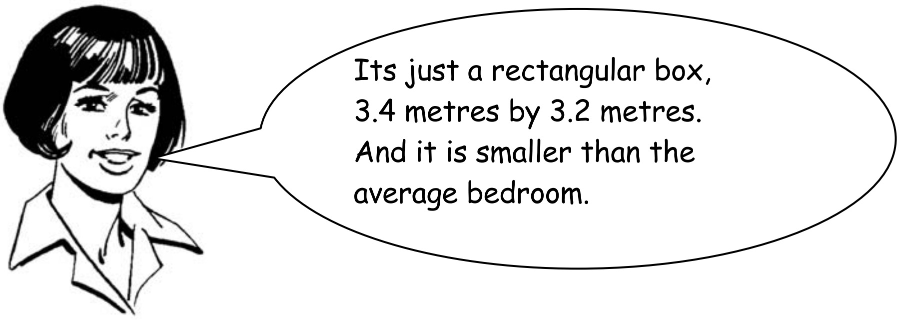
This is included to give practice at tackling a functional skills mathematics assessment question. As such it should take about 25 minutes to complete.

Part (a)	gives an opportunity for students to work with different units and to make a decision about size. Is it area to be considered – which average should be used? – mean, median or mode? or average length and average width? Make sure that students show/explain which average they have used
Part (b)(ii)	squared paper will probably be required. This part involves the use of scale in drawing.
Part (c)	in this part there are opportunities to consider dimensions of window frames and the height and width of the door. Calculations involving ratio will be needed here.

Main activity: Ellen's bedroom

Ellen and Annie are moving into a new flat. They both will have a study bedroom.

Ellen thinks her study bedroom is very small.



Annie disagrees and looks at some estate agents websites to see what the sizes of bedrooms seem to be. She makes this list to show Ellen:

4.4m by 3.4m	2.6m by 3.8m	3m by 2.7m
12 feet by 9 feet	13 feet by 14 feet	2.5m by 3.4m
9 feet by 13 feet	3.4m by 1.9m	11 feet by 10 feet
9.5 feet by 10 feet.		

She leaves Ellen to do any conversion of length, but tells her to use the fact that

1 foot = 0.3048 metres

(a) Is Ellen correct when she says her room is smaller than the average bedroom? Show how you decided.

Ellen decides to design the layout and furnish and paint the room herself.

She looks around the shops and on the internet and comes up with these items of furniture:

£65
Desk Code: AS/10
100W x 60D x 76H cm

£35
Product code 457
Seat 47w x 40d x 44h (cm)
Overall height 81 (cm)

•Dimensions (cm): Width: 90 Length: 190 Height: 56

Chelsea Divan Bed

£109.00

Product Information	
Width cm (in)	75cm (29.5")
Length cm (in)	190cm (74.8")
Order Code	SKU1609

The desk consists of two drawers, one niche, and a sliding shelf, in addition to a large work surface.

Dimensions: L96 x W45 x H88 cm.
Assembly required.

Full Price - £309.00
Sale Price NOW - £205.00

Rectangular Computer Desk AND Chair Quote: ER/03

Features

- Height 720mm
- 25mm-thick Desk Surface

Available in 4 Sizes
1800x800mm, 1600x800mm, 1400x800mm, 1200x800mm,

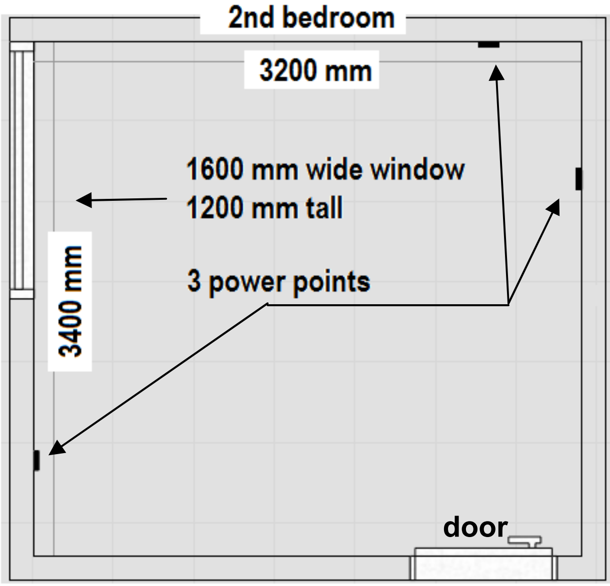
£210.00 SALE

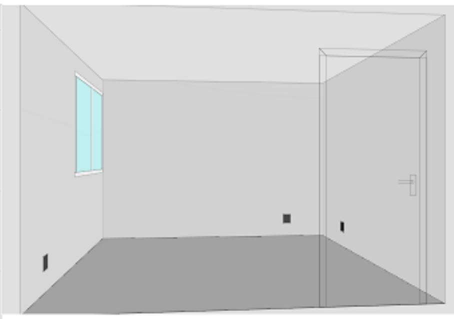
- Desk (H)76, (W)70, (D)54cm.
- Chair (H)77.5, (W)47, (D)47.5cm.
- Space saving desk and chair set.
- Chair weight 4.24kg.
- Total weight 18.8kg.
- Self assembly 1 person required.

Offer number: 785NG

£72.99

Here is a plan of Ellen's bedroom





Renshaw & Wren Architects

CADrawing No. 2754/05

(b)(i) **The most Ellen can spend on bed, desk and chair is £300. What would you advise her to buy?**

(b)(ii) **Draw a scale diagram of Ellen's new room showing where the bed, desk and chair fit in. (Choose a sensible scale)**

Ellen believes that sleeping and working in a room that is blue will be relaxing and the colour will help her study.

She finds this in a book about decorating:

Surface area which one litre of paint will cover:	
Gloss	14 m^2
Emulsion	12 m^2

These are prices of blue paint in her local DIY.

Blue emulsion (no under-coat needed)

5 litre cans £17.89

2.5 litre cans £9.98

Blue gloss (no under-coat needed)

1 litre cans £12

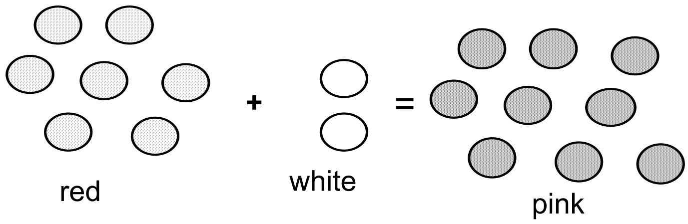
Ellen wants her room all blue; she will emulsion the walls, and gloss the window frame and doors.

(c) Estimate the cost of the paint she will need. Show all your calculations.

Examination and textbook type questions

A Paint

Pastel pink paint is made by mixing red and white paint in the ratio of 7 to 2



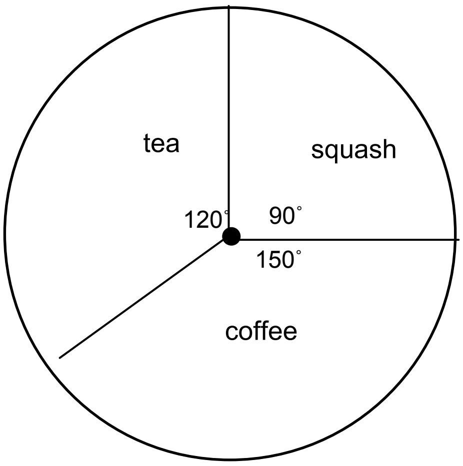
(a) Tom has 12 litres of white paint. How many litres of red paint does he need to make pastel pink paint?

(b) As part of an advertising campaign Paints - U - Like decide to paint a bridge pastel pink. They will need 450 litres of pastel pink paint.

How many litres of white paint are needed?

B Drinks

The pie chart represents the number of drinks sold at a school fête. A total of 720 drinks were sold. How many drinks of coffee were sold?



C Insurance

Mrs Smith insures the contents of her home for £4000. The cost is £1.20 for every £100 of cover. How much does the insurance cost?

D Maps

A popular scale for OS maps is 1: 50 000

(a) what does 1 cm on such a map represent on the ground?

(b) two schools are 6.4 cm apart on the map. How far apart are they on the ground?

E Coffee 1

A large jar of coffee costs £2.38 and a small jar costs 84p.

Express these prices as a ratio in its lowest terms

F Scale Drawing

A scale drawing of a playing field is drawn on a scale of 1:500

(a) what distance on the ground is represented by 6.3cm on the scale diagram?

(b) what distance on the scale drawing represents 82m on the ground?

G coffee 2

Here is a standard mix for making coffee:

4 cups of water 3 desert spoons of coffee

Decide which of these mixes makes coffee that is stronger than the standard mix

A	4 cups of water 5 desert spoons of coffee	B	3 cups of water 3 desert spoons of coffee
C	8 cups of water 7 desert spoons of coffee	D	40 cups of water 39 desert spoons of coffee
E	20 cups of water 12 desert spoons of coffee	F	2 cups of water 3 desert spoons of coffee
G	7 cups of water 6 desert spoons of coffee		

Questions like this can be adapted to raise other points – for example students could be asked to put the drinks in order of strength, starting with the weakest.

Further questions and ideas

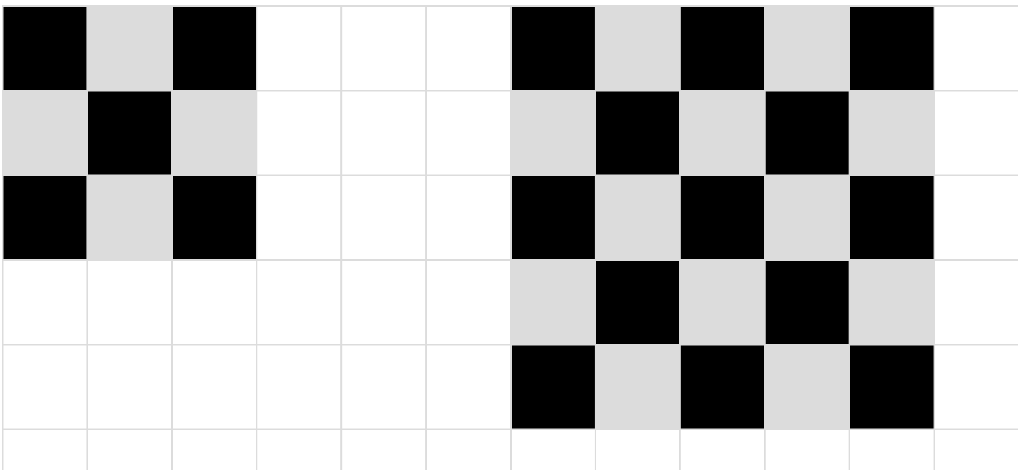
1 Which of these gives the sourest drink?

(a) A pure lemon juice and water drink mixed in the ratio 1:2
or
A lemon drink which is $\frac{1}{3}$rd pure lemon juice.

(b) A pure lemon juice and water drink mixed in the ratio 2:3
or
A lemon drink which is 20% pure lemon juice.

(c) a drink mixed with water to pure lemon juice in the ratio 3:5
or
a drink mixed with pure lemon juice to water in the ratio 5:2

2 questions about the proportion of grey to white tiles – is it increasing or decreasing as the pattern continues?



3 A challenge:

Solve this puzzle which comes from Ancient Greece.
"What number must be added to 100 and 20 so that the answers are in the ratio 3:1."
Make up and test some similar ones for yourselves.

4 Put these patterns into groups each having the same ratio of Xs to Os.

A: XXOOO B: OXXXO C: OXOXO
D: OXO E: XOO F: OOXXOO
G: XOXOX H: OOXXXOOXXX

5 Two stroke engines, like those used in petrol driven grass mowers or mopeds use a mixture of petrol and oil. The ratio, by volume of petrol to oil can be from 16:1 to 100:1 depending on the engine. Complete this table for different mixtures.

Petrol : oil ratio	millilitres of oil per litre of petrol (to the nearest whole number)
50:1	
60:1	
70:1	

OCR
RECOGNISING ACHIEVEMENT

Functional Skills Maths

Functional skills qualification in Maths at Level 1 and Level 2

Algebra

General Information

Introduction

It should be remembered that the algebraic demands of Level 1 and Level 2 range from National Curriculum levels 1 to 4 and 1 to 6 respectively. Given the very nature of mathematical functionality it is reasonable to assume that what is loosely termed algebraic manipulation will be less of a focus than is normally expected. The emphasis is more likely to be on using and selecting the algebra appropriate to solving a “real-life” task.

National Curriculum, GCSE Descriptors and Mathematics Functional Skills Criteria

National Curriculum descriptors relating to algebra for Levels 1 and 2

❶ recognise sequences of numbers, including odd and even numbers
❶ begin to use simple formulae expressed in words

❷ construct, express in symbolic form and use simple formulae involving one or two operations
❷ use brackets appropriately
❷ find and describe in words the rule for the next term or nth term of a sequence where the rule is linear
❷ formulate and solve linear equations with whole-number coefficients
❷ represent mappings expressed algebraically
❷ use Cartesian coordinates for graphical representation interpreting general features
❷ solve numerical problems and equations, use trial and improvement methods.

GCSE descriptors relating to algebra Foundation Tier

- distinguish the different roles played by letter symbols in algebra, using the correct notation
- distinguish in meaning between the words equation, formula, **identity** and expression
- manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors,
- set up and solve simple equations
- derive a formula, substitute numbers into a formula and change the subject of a formula
- solve linear inequalities in one variables, and represent the solution set on a number line
- use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
- generate terms of a sequence using term-to-term and position-to-term definitions of the sequence
- use linear expressions to describe the *n*th term of an arithmetic sequence
- use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

- recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
- construct linear functions from real-life problems and plot their corresponding graphs
- discuss, plot and interpret graphs (which may be non-linear) modelling real situations
- generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions.

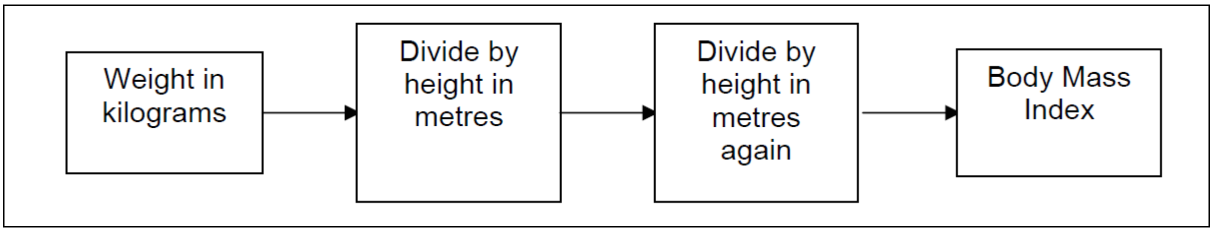
Level 1 and 2 Indicative content from Functional Skills Criteria

❶ use simple formulae expressed in words for one- or two-step operations

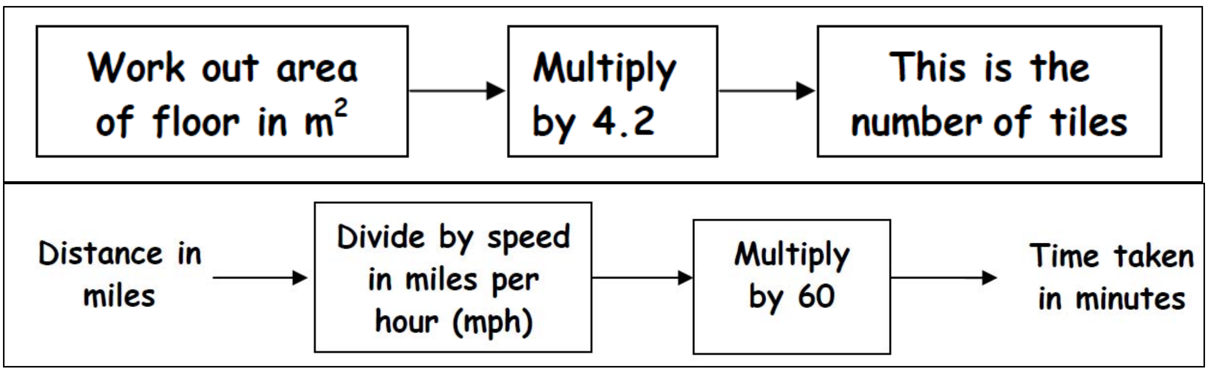
❷ understand and use simple formulae and equations involving one or two operations
❷ use information and communication technology (ICT) where appropriate

Experiences gained from the Functional Mathematics Pilot (2007 – 2010) Level 1

Questions involving algebraic techniques as defined above tend to be fairly well answered. They usually entail using some form number machine chain or flowchart, for example, to calculate a person's BMI:



Learners find the above accessible, with most errors a result of not converting heights in centimetres into one metres or of omitting one of the divisions. This could be interpreted as failing to follow the flowchart instructions complexly.
Question involving a more direct number machine such as those shown below, overall present no problems as such, although learners' carelessness over units can sometimes cause trouble.



Most errors appear to originate in not fully reading instructions on the number machine/flowchart – not in the underlying method of use.
Word equations, for example in the context of stretching springs have also been found accessible.

Extension = measured length – original length

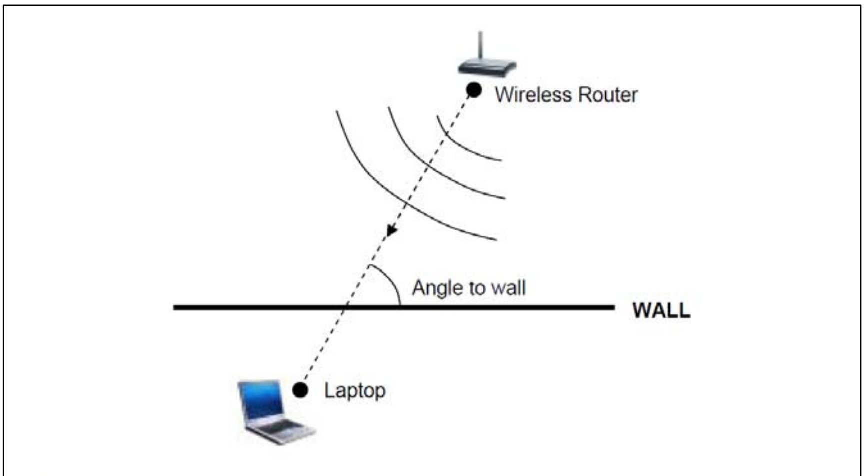
Level 2
As might be expected the algebraic content and general demand is increased in Level 2. However the emphasis is still on the use of formulae in practical situations. Such as ....

> **HOW TALL, HOW FAR AWAY?**
>
> This tells you how far away a tall building can be seen.
>
> **(1)** First find the square root of the building's height in metres.
>
> **(2)** Multiply this answer by 3.5
>
> **(3)** Now add 5.
>
> The final answer is how far away the building can be seen in kilometres.
>
> For example, a 36 m tall building can be seen from a distance of
>
> (6 x 3.5) + 5 = 26 km
>
> *(note: the square root of 36 is 6)*

or

> An **approximate value for the perimeter** of the same oval is worked out like this:
>
> (1) Find the value of a^2
> (2) Find the value of b^2
> (3) Add these two results
> (4) Take the square root of the answer
> (5) Multiply this by 4.
>
> This number is the approximate perimeter of the oval.

In common with Level 1 learners, Level 2 learners generally have little problem with the actual mechanics of inputting numbers into the word equation or number machine/flowchart. Most difficulties arise in selecting the appropriate number(s), especially in using the particular units as specified. In some cases learners would perhaps have been given some indication that an error had been made simply because of the unreasonableness of their answer. The area of checking both arithmetic and by the reasonableness or otherwise of the answer is one which few candidates – at any level – venture into.

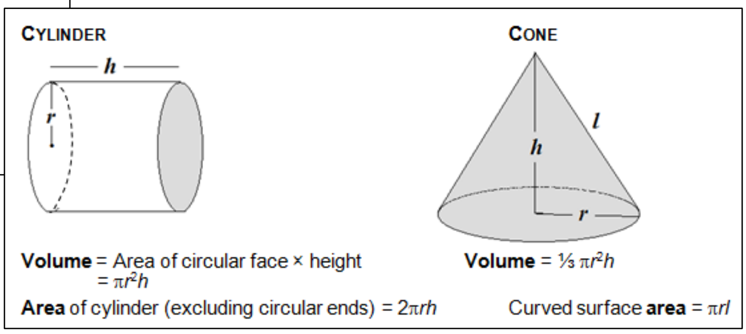


When a signal passes through two walls, the signal strength percentage received = (signal strength % through first wall) x (signal strength % through second wall)

Situations, such as that shown above, in which formulae have to be used embedded in less familiar contexts are found challenging by a large proportion of candidates.
Other siuautions where learners have to use a less familiar formula or select the appropriate one from a list, such as those below, tend to attract a fair number of errors.

Megan finds this formula, which gives the total amount of water, *W* cubic metres, collected a year from an average roof.
The area of the roof collecting water is *A* m^2 and the yearly rainfall *r* mm.

$$W = \frac{(r - 24) \times A}{1250}$$



To summarise whilst using word and algebraic formulae are not found difficult per se problems do arise when choosing the appropriate formula to use, using the correct units and in checking any results generated for arithmetic accuracy and reasonableness. To support the latter and as a general rule real-life numbers/results/measurements are used whenever feasible.

Common errors and misconceptions

The mathematics education literature is full of studies showing why algebra has acquired a reputation amongst teachers and learners alike, as one of the most difficult and troublesome aspects of secondary mathematics. However, this is not the place to present the results of a literature survey. Nevertheless, learners' difficulties in understanding of the symbols or letters used in algebra appear to have two main roots. In essence:

(1) different letters or symbols mean different numbers, ("*a* + *b* never equals *a* + *c*")

and (2) letters represent a shorthand for object names ("3a means 3 apples" rather than a representing the **number** of apples – a basic error which can still seen in some introductory texts and lessons. Many learners grow out of this erroneous initial model but for a significant proportion this remains the cornerstone on which their knowledge of algebra is built.)

There are other common errors (and taxonomies of them) such as “2x in line with fraction notation must represent 2 + x” or transferring previously learned place value knowledge “2x is the same as 20 + x”

Not surprisingly it has recently been highlighted that if learners are taught abstract ideas without meaning, this might not develop their understanding (and in many cases actually result in the reverse). In short if teachers want learners to “know” algebra then they must be given a deeper understanding and experience of the use of symbols.

The following activities are presented in the hope that they may address some of the issues indicated above.

The activities

In no particular order, a by no means exhaustive list of areas of weakness regularly generally observed are:

Reflecting on sensible nature of answers

Estimating Checking – both arithmetically and “is this a sensible answer?” Using consistent (and correct) units and stating them	***Possible remedy*** *Part and parcel of everyday teaching – “is that reasonable?” - make this and arithmetic checking second nature.* *In terms of “reasonableness” encourage scepticism, particularly in some text book (and exam) scenarios.* *Correct units and always putting units can probably only be achieved by constant reference during teaching – even the most capable tend to relapse into pure numbers rather than quantities.*

Making assumptions

Being confident enough to make them Stating them clearly and following them through	***Possible remedy*** *Can perhaps best be approached by questioning and contriving situations which force these issues (links with “Reflecting on sensible nature of answers) – a place here for starters such as “How many Maltesers will fill a room?”, “How many litres of water do you drink in a year?”, “How many exercise books do the secondary schools in the UK use in a year?” or “How fast do your nails/hair grow?” These all need the learner to make sensible assumptions and to “run” with them. These will also build-up learners’ confidence in making assumptions.*

Using the given data/information to carry out the task

Translating (internalising) the task Selecting appropriate data from a set with redundancy	***Possible remedy*** *Giving learners more experience in open tasks (this does not necessarily mean long tasks or in some cases actually doing them – scope perhaps for "virtual" how might you ..... giving practice in planning and communication, but not to be indulged in too often) – genuine whole-class discussion has a role here.* *Ensuring learners have to handle redundancy of information can be addressed by judicious choice of questions. This may require adapting current text book tests which tend to take a minimalist route.*

Communication

Presenting clear idea of passage through the task and sub-tasks. Making a clear conclusion(s) at appropriate points	***Possible remedy*** *Giving practice in completing similar tasks (the market is already responding i.e. SMP materials etc.) but more importantly set up situations where the learners (with appropriate sensitivity) read and listen to each other's work.* *This will make far clearer to learners issues of communication than thousands of cajoling words. (Building on (and finding out) about learners experiences/demands in English.)*

Some of the general points listed above are relevant to algebra and the following tasks.

Other useful sources

Some interesting overviews of learners' misconceptions in algebra:

http://www.learnquebec.ca/export/sites/learn/en/content/curriculum/mst/documents/algemisc.pdf

http://www.msri.org/communications/books/Book53/files/12foster.pdf

and of course

K M Hart (editor) Children's Understanding of Mathematics: 11-16 John Murray, London, 1981

Specific support for algebra in Support for Functional Skills Mathematics (SMP):

Level 1
20. Saving energy

Level 2
7. Fine Frames
11. New TV

Specific support for algebra in OCR GCSE in Mathematics A J562 AO3 Guide:
3.5 Case 5: Sequences
3.7 Case 7: Graphs

Too hot or too cold? (Level 1)

Introduction	This activity gives learners the opportunity to practice/revise using number machines.
Mode of working	Small group, with whole group plenary to discuss findings and conclusions.
Underpinning Knowledge	Some basic understanding of number machines, basic arithmetic involving whole numbers.
Materials	Calculators should be available for use when learners feel they need them.
Possible content	Using number machine chains in the context of word formula, working systematically, comparing results and drawing simple conclusions.
Functional skills *likely* to be used	❶ identify and obtain necessary information to tackle the problem ❶ select mathematics in an organised way to find solutions ❶ apply mathematics in an organised way to find solutions to straightforward practical problems for different purposes ❶ use appropriate checking procedures at each stage ❶ interpret and communicate solutions to practical problems, drawing simple conclusions and giving explanations ❷ understand routine and non-routine problems in familiar and unfamiliar contexts and situations ❷ draw conclusions and provide mathematical justifications
Starter	The starter should bring to light any serious misconceptions regarding using number machines and allow for remediation.
Plenary/homework	Learners could be asked to find some word formulae used in real-life, for example, distance away from thunder storm by counting, rough DIY formulae eg amount of sand = 5 x amount of cement etc. There will be opportunities here to discuss issues of consistent units – indeed this way well arise naturally as in the illustration here which is devoid of any units!

Starter Activity: Too hot or too cold?

Preamble: This is a straight forward activity whose purpose is to give learners the opportunity to re–cap using number machines. This should pave the way for subsequent work on using number machines in the context of formulae. The activity could begin as an individual/pair activity.

Content and processes: Basic arithmetic, working systematically.

Equipment: Possibly calculators.

Answers: Learners should check each other's answers.

Extensions: Experimenting to find an arrangement giving the same output as input.

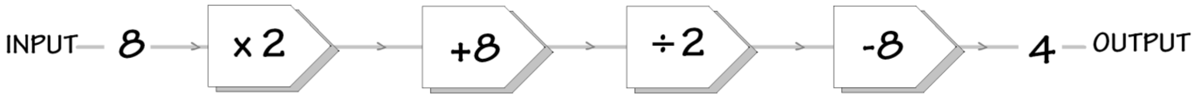
Main Activity: Too hot or too cold?

Preamble:	With this activity learners need to make some initial decisions. Firstly, what, very roughly, is room temperature and what are the units? Secondly, which way to convert? It is making these choices and acting on them that makes the activity functional. There is obvious scope for discussion about these choices. A number of learners will need some initial support.
Content and processes:	Using word equations in order to solve a problem.
Equipment:	Possibly calculators (spreadsheet for some more capable Level 2 learners).
Answers:	Learners should check their answers.
Extensions:	Investigate to see how the difference between accurate and approximate formulae conversions changes with temperature. With some Level 2 learners the use of a spreadsheet might be appropriate. Other exact and approximate conversions are litres to pints (x 1.76 and [x 7 → ÷ 4]) and miles per hour to km per hour (x 1.61 and [x 8 → ÷ 5]).

Starter Activity: Too hot or too cold?

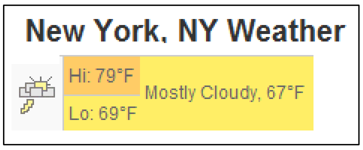
This number machine chain has an INPUT of 8 and an OUTPUT of 4.

How many different OUTPUTS can you find for an
INPUT of 8 by re-arranging these number machines?



Main Activity: Too hot or too cold?

Temperatures used to be measured using the Fahrenheit scale (°F).
This is still used in the US.
Most of the world now uses the Celsius scale (°C).



These two formulas convert exactly between the two temperature scales.

To convert from

°F to °C

Temperature in °F → - 32 → x5 → ÷9 → Temperature in °C

and

To convert from °C to °F

Temperature in °C → ×9 → ÷5 → +32 → Temperature in °F

There are also two rough rules which are simpler.

To change from °F to °C: double the °F temperature and add 30.

To change from °C to °F: take away 30 from the °C temperature and divide by 2.

How close in the rough rule to the exact rule for temperatures round about room temperature.

What's it worth? (Level 2)

Introduction	This activity gives learners the opportunity to practice/revise finding the numerical value of symbols in a puzzle situation.
Mode of working	Small group, with whole group plenary to share ideas.
Underpinning Knowledge	Basic arithmetic involving whole numbers.
Materials	Calculators should be available for use when learners feel they need them.
Possible content	Finding the numerical values of symbols informally.
Functional skills *likely* to be used	❶ apply mathematics in an organised way to find solutions to straightforward practical problems for different purposes ❶ use appropriate checking procedures at each stage ❷ draw conclusions and provide mathematical justifications ❷ choose from a range of mathematics to find solutions ❷ draw conclusions and provide mathematical justifications
Starter	The starter forces into relief the misconception of symbols representing the same number and the need to qualify this in order to solve standard "find the number problems". It may also provoke discussion regarding the "equal sign" what it represents. Discussion is a crucial ingredient of this activity.
Plenary/homework	Producing more puzzles of their own in continuation of the activity.

Starter Activity: What's it worth?

Preamble: A very standard activity but one whose purpose here is to bring to the fore
some common "pre-algebra" errors.

Content and processes: Basic arithmetic, informal solution of puzzles set in equation-like scenarios.

Equipment: Just possibly but unlikely calculators.

Answers: The learners own solutions and subsequent discussions.

Extensions: The logical extension of this activity follows on in the activity proper.

Main Activity: What's it worth?

Preamble:	A short activity which allows learners to experiment informally constructing different expressions and solving (and checking the solutions) in the subsequent puzzles. Encourage learners to be inventive and be prepared to moderate possible arguments. Although in essence simple the activity does give learners the opportunity, at an informal level to "discover" some of the basic structures of algebra.
Content and processes:	Basic arithmetic, informal solution of puzzles set in equation-like scenarios.
Equipment:	Possibly calculators.
Answers:	The learners own puzzles/solutions and subsequent discussions.
Extensions:	Depending on circumstances further puzzles could be constructed (and solved) as a homework task. There is obvious scope here for display work.

Starter Activity: What's it worth?

Jot down all you can find out about the numbers in the boxes.

(a) ☐ + 4 = 10

(b) 8 + 5 = ☐ + 4

(c) 10 + ☐ = 10 + ☐

(d) 10 + ☐ = 5 + ☐

(e) ☐ x ☐ = 36

Main Activity: What's it worth?

Working in pairs make up and test on some other pairs puzzles like the ones you have just been solving. Use some symbols of your own for the unknown numbers.
Be inventive!

Diet Fads (Level 2)

Introduction	The aim of this activity is to give learners the opportunity to practice their algebraic skills within a believable context using spreadsheets as the vehicle. Without the use of a spreadsheet the activity the activity shifts to one involving informal trial and improvement. The later approach is more suitable for Level 1 learners. Evidence may be found in the literature that constructing spreadsheets and the practice it gives in using symbols can have a positive effect on learners' algebraic understanding.
Mode of working	Pairs working together, with whole-group sharing of ideas in a plenary.
Underpinning Knowledge	Some knowledge of the use and application of spreadsheets or reasonable confidence with basic arithmetic.
Materials	Computer loaded with spreadsheet (most probably EXCEL or an "open software" equivalent). A calculator for checking that the spreadsheet is error free.
Possible content	Using algebraic and numeracy skills in the construction and testing of formulae based on standard elementary spreadsheet functions.
Functional skills *likely* to be used	❶ understand practical problems in familiar and unfamiliar contexts and situations, some of which are non-routine ❶ use appropriate checking procedures at each stage ❶ select mathematics in an organised way to find solutions ❶ interpret and communicate solutions to practical problems, drawing simple conclusions and giving explanations ❷ understand routine and non-routine problems in familiar and unfamiliar contexts and situations ❷ identify the situation or problems and identify the mathematical methods needed to solve them ❷ use appropriate checking procedures and evaluate their effectiveness at each stage ❷ interpret and communicate solutions to multistage practical problems in familiar and unfamiliar contexts and situations
Starter	The starter touches upon some of the algebraic skills necessary for the main activity.
Plenary/homework	With the starter other ways of scoring in sport could be considered, for example points for gold, silver and bronze medals (traditionally 3, 2, 1 but a case could be made for 4, 2, 1 – would this alter the rankings etc) – allow learners to decide (and express algebraically their own approaches – perhaps comparing them with the 3, 2, 1 model). Other scoring systems could be touched upon F1 (from) 2010 : 12, 9, 7 etc. from 10, 8, 6 etc., go karting: 30, 25, 21, 18, 16 or rugby. In each case the occurrence or not of impossible scores could be investigated. A spreadsheet would be useful adjunct to investigate changes in total points and even changes in ordering (the latter only requires use of the ordering button).

Starter Activity: Diet Fads

Preamble:	Practice setting up a simple algebraic formula in the context of a points score, encourage learners to check their formulae using some numbers of their own 0- depending on circumstances as useful issue could be made of this as a method of checking formulae – try inputting some simple numbers and so if the “work”.
Content and processes:	Basic algebra, but hopefully using a context which will engage and also one where the answer can be easy checked by the learner (perhaps after some support in this direction).
Equipment:	None.
Answers:	The learners own solutions and subsequent discussions.
Extensions:	See above.

Main Activity: Diet Fads

Preamble:	See beginning section.
Content and processes:	Dependent on Level (see above)
Equipment:	See above.
Answers:	There is no definitive answer.
Extensions:	See above.

Starter Activity

In most sports points are awarded for winning, drawing or finishing first, second, third etc.

In some basketball leagues a win is awarded 3 points, a draw 1 point and a lose nothing.

Write down the formula giving the score, *S*, for a team that wins *w* matches, draws *d* matches and loses *l* matches.

How many different results can you find that give a total of 10 points?

Main Activity: Diet Fads (Level 2)

Jim is always trying new diets. He finds this one on the Internet.

DON'T GO BANANAS OVER YOUR DIET TRY THE BANANA AND MILK DIET.
ITS SIMPLE YOU JUST EAT BANANAS AND MILK!!

This table shows the nutritional value of bananas and milk.

	Calories	**Grams of protein**	**Grams of carbohydrate**	**Milligrams of vitamin C**
1 kg of bananas	960	15	240	105
1 litre of milk	640	35	50	0

According to medical advice site on the Internet someone of Jim's age must have at least 1920 calories, 60g of protein and 70mg of vitamin C a day.

Use a spreadsheet to find the mix which will give Jim the least carbohydrate.

Main Activity: Diet Fads (Level 1)

Jim is always trying new diets. He finds this one on the Internet.

DON'T GO BANANAS OVER YOUR DIET TRY THE BANANA AND MILK DIET.
ITS SIMPLE YOU JUST EAT BANANAS AND MILK!!

This table shows the nutritional value of bananas and milk.

	Calories	**Grams of protein**	**Grams of carbohydrate**	**Milligrams of vitamin C**
1 banana	100	1	30	0.01
1 glass of milk	150	10	15	0

According to medical advice site on the Internet someone of Jim's age must have at least 2000 calories but less than 2500 calories, about 60g of protein and some vitamin C a day.

Springs (Level 2)

Introduction	The main activity gives learners the opportunity to use algebraic skills within a believable context using spreadsheets as the vehicle.
Mode of working	Pairs working together, with whole-group sharing of ideas in a plenary.
Underpinning Knowledge	Some knowledge of the use and application of spreadsheets.
Materials	Computer loaded with spreadsheet (most probably EXCEL or an “open software” equivalent). A calculator for checking that the spreadsheet is error free.
Possible content	Use numeric skills in the construction and testing of formulae based on standard elementary spreadsheet functions. Use and generate formulae. Substitute numbers into a formula or an expression, checking formulae by substituting. (Using the SUM function to sum deviations in some form). Informal trial and improvement.
Functional skills *likely* to be used	❷ understand routine and non-routine problems in familiar and unfamiliar contexts and situations ❷ identify the situation or problems and identify the mathematical methods needed to solve them ❷ use appropriate checking procedures and evaluate their effectiveness at each stage ❷ interpret and communicate solutions to multistage practical problems in familiar and unfamiliar contexts and situations ❷ draw conclusions and provide mathematical justifications
Starter	The starter, about making a spring, is not directly related to the algebra but more numerical in nature but involves constructing a spring so may support learners’’ appreciation of the scenario.
Plenary/homework	Depending on circumstances learners could test their estimates and actually make a spring!

Starter Activity: Springs

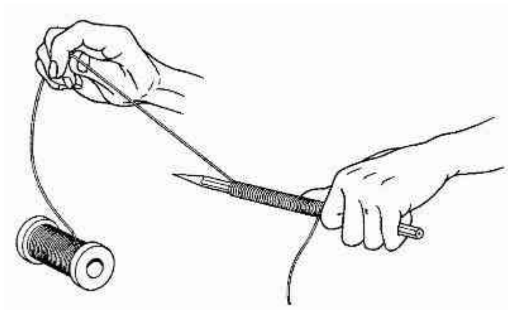
Preamble:	This starter requires learners to make a series of assumptions (wire diameter, length of spring/pencil, possible use of circle formula etc.). Some learners may need some initial support and guided to realise that all that is required in sensible choices, but otherwise “anything goes” subject to the context of the question. The activity is suitable for whole-group or small group or even individual work, but with the proviso that to gain full benefit learners must be able to discuss their strategies and reasoning behind their estimates.
Content and processes:	Basic arithmetic skills, the ability to make reasonable estimates, possible use of the circle circumference formula.
Equipment:	None – unless it is feasible to actually make the spring.
Answers:	The learners own estimations.
Extensions:	See above.

Main Activity: Springs

Preamble:	An activity in which learners need to decide how to measure “goodness of fit”. Ideally this should emerge from learner/learner discussion with the minimum amount of intervention.
Content and processes:	Basic arithmetic, constructing a spreadsheet, using of basic spreadsheet functions.
Equipment:	Computers loaded with standard spreadsheet (most probably EXCEL or an “open software” equivalent, possibly a calculator to check spreadsheet formulae).
Answers:	The learners own solutions and subsequent discussions, $a = 6.5$ gives a better fit.
Extensions:	None.

Starter Activity: Springs

Springs
You can make a spring by winding wire round a pencil.

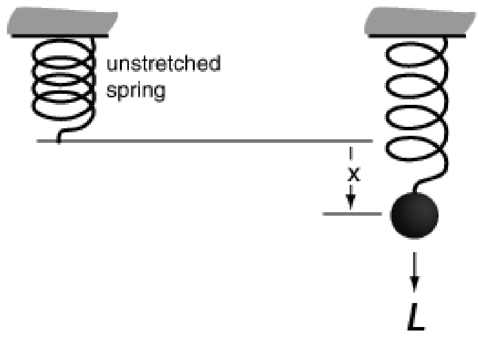


About how much wire would you need to make such a spring?

Jot down all the assumptions you make.

Main Activity: Springs

Some students are investigating how the distance stretched by a spring (*x* cm) changes with the load *L* newtons.



Here are their results.

L (N)	2	3	4	5	6	7	8	9	11	12	15	18
x (cm)	0.3	0.5	0.6	0.8	0.9	1	1.3	1.3	1.7	2	2.2	2.8

Jan says the equation connecting *L* and *x* is $L = ax$, where x is a constant (does not change with *L* or *a*) whose value is 6.7, so $L = 6.7x$.

Is she right? Can you find a better value for *a* and **show** that it is better?
Support your answer with some numbers; using a spreadsheet will help you.

What’s it all mean? (Level 2)

Introduction	The main activity gives learners the opportunity to practice using numbers to represent letters in a concrete situation. The vehicle – dice games – is hardly new but is has the advantage that it rapidly brings to the fore errors associated with “letters as abbreviations” or other common errors as well; as well as giving a moderately realistic scenario in which to apply algebra.
Mode of working	Pairs working together, with whole-group sharing of ideas in a plenary. Pairs exchanging their winning rules should produce some fruitful discussion, especially if inequalities are used.
Underpinning Knowledge	Some previous exposure of the use of letters to represent quantities. An understanding of the symbols > and <.
Materials	Dice, black and red – for learners to actual play and test their algebraic rules.
Possible content	Using letters to represent numbers. Substitute numbers into an expression, checking formulae by substituting.
Functional skills *likely* to be used	❷ understand routine and non-routine problems in familiar and unfamiliar contexts and situations ❷ identify the situation or problems and identify the mathematical methods needed to solve them ❷ interpret and communicate solutions to multistage practical problems in familiar and unfamiliar contexts and situations ❷ draw conclusions and provide mathematical justifications
Starter (s)	The starter, which is challenging, is intended to give learners the opportunity to think algebraically – some will almost certainly need initial support. However by challenging them in the starter and solving major problems as a result, it will ease any possible difficulties with the main activity. A collection of possible additional starters is included at the end. They are broadly in order of difficulty – with some of the later ones tending towards pure algebra. Most are useful to stimulate discussion after some initial work either as individuals or as pairs. In almost all of them learners could be encouraged to make up similar problems and test them for themselves.
Plenary/homework	Discussion and investigation of “impossible” rules can be a fruitful activity.

Starter Activity: What's it all mean?

Preamble: This starter gives learners opportunities to consider the meaning of some simple algebraic expressions using the number line as support. Without doubt a proportion of learners will need some initial support. However, the discussion resulting, should in the long term advance learners understanding of algebraic symbolism.

Content and processes: Interpreting algebraic expressions and explain these verbally.

Equipment: None.

Answers: The learners own (correct) statements – for example

a) a negative number which is the difference between s and r.
b) a positive number which is the difference between s and r.
c) the mean of a, b and c.
d) a positive number, difference between twice s and r.
e) half the difference between s and r, a positive number.
f) t divided by s , a number greater than 1.
g) r divided by s , a number less than 1.
h) a point half way between s and t, a positive number.

Extensions: In some circumstances learners could be invited to make up and test on each other similar expressions.

Starter Activity 2: What's it all mean?

Preamble: This activity gives learners practice in forming and looking closely at algebraic expressions.

Content and processes: Forming and evaluating simple algebraic expressions subject to conditions. Depending on circumstances using informal elementary ideas of probability / fairness. Making lists systematically.

Equipment: Blue and red dice – if learners are to play the games

Answers: r/b = 1/4 , 2/3, 3/2, 4/1
A score of $2r + b = 20$ is impossible to get with 1 – 6 dice.

Extensions: Going into more detail on the probability of winning and listing of all possible outcomes.

Starter Activity 3: What's it all mean?

Preamble: This activity draws out the common errors made when learners form equations where the variables are linked by a multiplicative constant (which is to write the latter the "wrong way round").
It is a good point at which to stress that, for example, m (in $3m = f$) represents the number of metres (not "metres") and f the number of feet (not "feet").

Content and processes: Checking algebraic expressions.

Equipment: None.

Answers: $3m = f$ (3 feet in a metre) m = number of metres, f = number of feet

Extensions: Ask learners to find and write equivalent expressions for other conversions.

Starter Activity 4: What's it all mean?

Preamble: More drawing out possible common errors.

Content and processes: Focusing on the error "symbol as abbreviation" rather than number.

Equipment: None.

Answers: $3s + 2a$ gives the total points Amber scored
x/y = 0/20, 1/19, 2/18 ...... 10/10 ....

Extensions: Give learners more complex equations and encourage them to be inventive as to the situations they might describe. (eg $W = a \div b$ etc.)

Starter Activity 5: What's it all mean?

Preamble: A fairly standard activity, to encourage discussion.

Content and processes: Recognising that letters may represent the same number, interpreting algebraic expressions linked by "=".

Equipment: None.

Answers: (1) Sometimes (2 and 2) (2) Never (3) Sometimes (when $d = e$)
(4) Always true (5) Sometimes (when $g = 1$)
(6) Sometimes (when $o = p$)

Extensions: Invite learners to make up and test on each other similar questions.

Starter Activity 6: What's it all mean?

Preamble: A purely algebraic activity, which should encourages discussion.

Content and processes: Recognising that letters may themselves represent other letters, informally manipulate simple algebraic expressions.

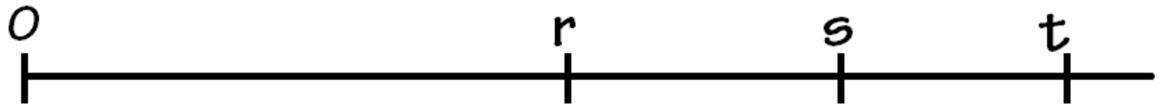
Equipment: None.

Answers: (1) True (2) Not true (= 2x) (3) True
(4) Not true (5) True (6) True.

Extensions: Invite learners to make up and test on each other similar questions.

Starter Activity 1: What's it all mean?

This is a number line, a, b and c are points on it.



Jot down all you can work out about the meaning or values of:

(a) $s - r$ (b) $r – s$

(c) $(r + s + t)÷3$ (d) $2s – r$

(e) $(s – r) ÷ 2$ (f) $t ÷ s$

(g) $r ÷ s$ (h) $s + (t - s) ÷ 2$

Starter Activity 2: What's it all mean?

Algebra can be used to summarise rules.
For example, imagine playing a game with a red and a blue dice.

If the winning score is one where the numbers on the red dice and blue dice added is 5,

the rule for the winning score could written as $r + b = 5$

Where r represents the number on the red dice
b represents the number on the blue dice

Complete this list all the possible winning throws

r =	1	2	...
b =	4	3	...

What's wrong with having a winning rule 2r + b = 20?

Make up some winning rules for yourselves – make them interesting – perhaps use the symbols > and < rather than =. If you can test them out – which are fair?

Starter Activity 3: What's it all mean?

Sometimes it is useful to be able to convert between different units of measurement.
Use the information in the box to help work out what the letters might represent in some of these formulas.

- Three feet are about one metre.
- An ounce is about 25 grams
- An inch is 254 millimetres

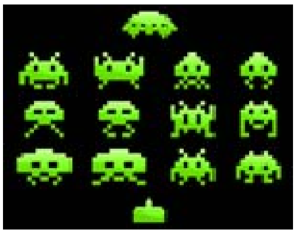
$m = 3f$
$w = 25m$
$25m = w$
$a=25b$
$3m = f$

Starter Activity 4: What's it all mean?

- In a video game, destroying a flying saucer is worth *s* points, and destroying an alien worth *a* points.
  Amber destroys 3 rockets and 2 aliens.
  What does 3s + 2a represent?

- Jot down all you can about x and y which both represent whole numbers, if $x + y = 20$.

- Try to write a situation/story in a couple of sentences which can be represented by $D=a + 3b$.



Starter Activity 5: What's it all mean?

Always, sometimes (if so when?) or never true?
You decide, but be prepared to support your answer!

1) $a + b = ab$
2) $c + 10 = c + 12$
3) $\boldsymbol{d} + \mathbf{12} = \boldsymbol{e} + \mathbf{12}$
4) $\mathbf{2}\boldsymbol{f} + \mathbf{3} = \mathbf{3} + \mathbf{2}\boldsymbol{f}$
5) $\mathbf{3} + \mathbf{2}\boldsymbol{g} = \mathbf{5}\boldsymbol{g}$
6) $m + n + o = m + n + p$

Starter Activity 6: What's it all mean?

When $x = 3a$ and $2b = x$, which are these are true and which are not true?

You will need to be able to explain why!

(1)	$3a = 2b$	(2)	$3a + 2b = x$	(3)	$2x = 3a + 2b$
(4)	a is bigger than b	(5)	$3x = 3a + 4b$	(6)	$6a – 2b = x$

OCR Customer contact centre

VOCATIONAL QUALIFICATIONS
Telephone 024 76 851509
Facsimile 024 76 851633
Email vocational.qualifications@ocr.org.uk

GENERAL QUALIFICATIONS
Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk

PART OF THE CAMBRIDGE ASSESSMENT GROUP

FS 27093