Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate in Mathematics for Engineering **H860/02**

Paper 2

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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Question			Answer	Marks
1	(a)		Both tables show a final velocity of 15.59 m/s for the 2 m parachute. Since the object in the second table has travelled 200 m further than in the first table without an increase in velocity, it is reasonable to assume that 15.59 m/s is approximately the terminal velocity. The same argument cannot be applied to the 1 m and 1.5 m parachutes. However, the two final velocity values for the 1.5 m parachute are very close and it would be reasonable to conclude that the terminal velocity is very little more than 27.66 m/s. The terminal velocity for the 1 m parachute is inconclusive.	2 3 **[5]**
	(b)		k can be deduced directly from $\frac{d^2x}{dt^2} = g - kS\frac{dx}{dt}$ when $\frac{d^2x}{dt^2} = 0$ and $\frac{dx}{dt}$ is known. Using the fact that the 2 m parachute provides a terminal velocity of 15.59 m/s $g - k \times \pi \times 15.59 = 0$ $k = g/(\pi \times 15.59) = 9.8/(3.14159 \times 15.59) = 0.2$	2 2 **[4]**
			Total	**[9]**

Question			Answer	Marks
2	(a)	(i)	$\frac{d^2h}{dt^2} = -g$ (acceleration)	1
		(ii)	$\frac{dh}{dt} = -gt + V_0$ where V_0 is the initial upward velocity	2
		(iii)	$h = \frac{-gt^2}{2} + V_0 t + A_0$ where A_0 is the initial height (= 0)	2 **[5]**
	(b)		Maximum height is reached when $\frac{dh}{dt} = 0$ $-gt + V_0 = 0 \qquad t = \frac{V_0}{g}$ Maximum height = 200 $\frac{-gt^2}{2} + V_0 t = 200$ $\frac{-g}{2}\left(\frac{V_0}{g}\right)^2 + V_0\left(\frac{V_0}{g}\right) = 200$ $-\frac{V_0^2}{2} + V_0^2 = 200g$ $V_0^2 = 400g$ $V_0 = \sqrt{400g} = 62.61 \text{ m s}^{-1}$	1 1 1 1 1 **[5]**

			Alternative acceptable solution (final velocity)^2 = (initial velocity)^2 – 2gs $0 = u^2 - 2gs$ $u = \sqrt{2gs} = \sqrt{2\times 9.8\times 200} = 62.61 \text{ m s}^{-1}$	
			Total	**[10]**

Question			**Answer**	**Marks**
3	**(a)**	**(i)**	$\frac{d^2x}{dt^2}$ is acceleration i.e. the rate of change of velocity v wrt t	
			i.e. $\frac{d^2x}{dt^2} = \frac{dv}{dt}$	1
			$\frac{dx}{dt} = v$ velocity	
			Replacing in $\frac{d^2x}{dt^2} = g - kS\frac{dx}{dt}$ gives	
			$\frac{dv}{dt} = g - kSv$	1
				[2]
		(ii)	Using integrating factor method : $\frac{dv}{dt} + Pv = Q$	
			$P = kS \qquad Q = g \qquad IFv = \int Q.IFdt$	1
			$IF = e^{\int Pdt} = e^{\int kSdt} = e^{kSt}$	
			$e^{kSt}v = \int ge^{kSt}dt$	1
			$e^{kSt}v = \frac{g}{kS}e^{kSt} + A$	1
			$v = \frac{g}{kS} + Ae^{-kSt}$	1
			when $t = 0, v = 0 \Rightarrow A = -\frac{g}{kS}$	1
			$v = \frac{g}{kS} - \frac{g}{kS}e^{-kSt}$	1 **[6]**
			$v = \frac{g}{kS}(1 - e^{-kSt})$	
			Alternative acceptable solution using separation of variables	
			$\frac{dv}{dt} = g - ksv$	
			$\int \frac{dv}{g - ksv} = \int dt$	
			$-\frac{1}{ks}\ln(g - ksv) = t + C$	

			$\ln(g - ksv) = -kst + B$ $g - ksv = Ae^{-kst}$ $ksv = g - Ae^{-kst}$ $v = \frac{g}{sk} - De^{-kst}$ when $t = 0, v = 0 \Rightarrow D = \frac{g}{kS}$ $v = \frac{g}{kS}(1 - e^{-kSt})$	
	(b)		$v = \frac{g}{kS}(1 - e^{-kSt})$ $x = \int v\mathrm{d}t$ $x = \int \frac{g}{kS}(1 - e^{-kSt})\mathrm{d}t$ $x = \frac{g}{kS}\left(t + \frac{e^{-kSt}}{kS}\right) + B$ When $t = 0$ $x = 0$ $0 = \frac{g}{kS}\left(\frac{1}{kS}\right) + B \Rightarrow B = -\frac{g}{(kS)^2}$ $x = \frac{g}{kS}\left(t + \frac{e^{-kat}}{kS}\right) - \frac{g}{(kS)^2}$ $x = \frac{g}{kS}\left(t + \frac{1}{kS}(e^{-kSt} - 1)\right)$	1 1 1 1 **[4]**
			Total	**[12]**

Question			**Answer**	**Marks**
4	**(a)**		Using $v = \frac{g}{kS}(1 - e^{-kSt})$ Terminal velocity reached as $t \rightarrow \infty$ i.e. as $e^{-kSt} \rightarrow 0$ $v = \frac{g}{kS}$ Also allow use of $\frac{\mathrm{d}v}{\mathrm{d}t} = g - kSv$ from Q3a i.e. $0 = g - ksv$	1 **[1]**
	(b)	**(i)**	$S = \frac{g}{5k} = \pi\left(\frac{d}{2}\right)^2$ $d = 2\sqrt{\frac{g}{5k\pi}}$ $d = 2\sqrt{\frac{9.8}{5 * 0.25\pi}} = 3.16$ m	3 **[3]**

		(ii)	Half terminal velocity		
			$\frac{g}{Sk}(1-e^{-kSt}) = 2.5$	1	
			$(1-e^{-kSt}) = \frac{2.5Sk}{g}$	1	
			$e^{-kSt} = 1-\frac{2.5kS}{g}$		
			$-kSt = \ln\left(1-\frac{2.5kS}{g}\right)$	1	
			$t = \frac{-\ln\left(1-\frac{2.5kS}{g}\right)}{kS}$	1	
			When $k = 0.25$,		
			$S = \pi\left(\frac{d}{2}\right)^2 = 7.84$	1	**[5]**
			$t = 0.353647$ s		
			Total		**[9]**

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